GENERALIZED K-T BLAST AND K-T SENSE USING FOCUSS

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ABSTRACT

According to the recent theory of compressed sensing, accurate reconstruction is possible even from data samples dramatically smaller than Nyquist sampling limit as long as the unknown image is sparse. In MRI, dynamic imaging such as cardiac cine may be a nice application of the compressed sensing theory since it requires significant reduction of data acquisition time whereas the periodic motions are usually sparse in spectral domain. The main contribution of this paper is to show that a sparse reconstruction method called the FOCal Underdetermined System Solver (FOCUSS) is a very effective compressed sensing reconstruction algorithm by exploiting the sparsity in spectral domain. Furthermore, our analysis reveals that celebrated k-t BLAST and k-t SENSE are special cases of our algorithm; hence our algorithm outperforms them in general situations.

1. INTRODUCTION

Dynamic MRI is a technique to monitor dynamic processes such as cardiac motion. To improve the temporal resolution, many methods have been considered. Fast imaging sequences like Echo-planar imaging (EPI), turbo spin echo (TSE), spoiled gradient methods like fast low angle shot (FLASH), and stead stead free precession (SSFP) have been widely used; and parallel imaging methods like SENSE, SMASH, and GRAPPA have been incorporated to reduce scan time even further by skipping the phase encoding steps [1]. Current state of art SSFP cine gradient echo (GRE) techniques can reconstruct 150×256 matrix within 7-10 seconds using True TR=2ms and TE=1ms [1]. Another seemingly different approach called k-t BLAST and k-t SENSE [2] have been proposed by taking advantage of *a priori* information from the training data set. In k-t BLAST/SENSE, even if the spectral supports are overlapped due to aliasing, *a priori* information from training data can be used to remedy the aliasing artifacts. With the help of k-t BLAST/SENSE, a multiple slice of cardiac cine can be obtained within a single breath hold [2].

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Recently, in signal processing community, there has been extensive research investigation about so called "compressed sensing" theory [3, 4]. The compressed sensing theory tells us that perfect reconstruction is possible even from the samples dramatically smaller than the Nyquist sampling limit. Furthermore, even if the signal is not perfectly sparse, the compressed sensing can still recover the significant features of the signals as long as the signal is compressible. One of the significant contributions of the compressed sensing theory is that, under some condition on the sample size, the solutions from computationally feasible algorithms such as the basis pursuit, matching pursuit methods, or the convex L_1 optimization method are equivalent from the solutions from the computationally prohibited original sparse reconstruction problem [3, 4]. Hence, the compressed sensing theory has a great potential for imaging problems.

Interestingly, these seemingly different k-t BLAST/SENSE and compressed sensing algorithms are very closely related to each other. Through this paper, we will clearly show the connection between them. More specifically, this paper shows that the celebrated k-t BLAST/SENSE is equivalent to a special cases of our new algorithm called k-t FOCUSS (k-t space FOCal Underdetermined System Solver)[5] derived from compressed sensing theory. FOCUSS was originally designed to obtain sparse solutions by successively solving quadratic optimization problems [5]. More specifically, FOCUSS starts by finding a low resolution estimate of the sparse signal, and then, this solution is pruned to a sparse signal representation. The pruning process is implemented by scaling the entries of the current solution by those of the solutions of previous iterations. It turns out that the low resolution images of k-t BLAST/SENSE obtained from the training data, are the initial low resolution images required for the convergence of FOCUSS, and the k-t BLAST and k-t SENSE are exactly matched to the first iteration of the k-t FOCUSS algorithm. Such a finding reveals that more k-t FOCUSS iteration may improve the reconstruction image quality of k-t BLAST and k-t SENSE significantly. This is important in clinical applications since the even with the advances of ultra fast cine GRE pulse sequences, much high spatial resolution, such as 1-2mm, would help to define finer structures such as cardiac valves, or smaller vessels such as coronary ateries or bypass grafts [1].

FOCUSS is nicely fitted to the dynamic MRI like cardiac motion. First, the initial low resolution images essential for FOCUSS can be easily obtained from the data. For example, in the cartesian trajectory, when the random sampling patterns are used with more samples at low frequency region, the initial low-resolution estimate can be obtained without training data set. For the cases of radial trajectory, the initial low resolution image can be obtained from over-sampled center region in k-space. Second, FOCUSS incorporates the sparseness as a soft constraint, hence the resultant image is visually pleasant compared to other L_1 minimization algorithms. Third, FOCUSS can be very easily implemented in a computationally efficient manner using successive quadratic optimization. These are quite big advantages over the other sparse optimization algorithms such as basis pursuit or matching pursuit approaches. Finally, as iteration goes, the FOCUSS asymptotically achieves the optimal solution from the compressed sensing point of view since FOCUSS minimize the L_1 cost function. Experimental results demonstrate very quick convergence of the k-t FOCUSS to accurate solutions even from highly sparse k-space samples. Also, the results show that k-t FOCUSS is a generalization of k-t BLAST/ SENSE and outperforms them.

2. REVIEW OF K-T BLAST AND K-T SENSE

In dynamic MR imaging, the temporal resolution is important as well as the spatial resolution. The k-t BLAST (Broad-use Linear Acquisition Speed-up Technique) and k-t SENSE (SENSitivity Encoding) have demonstrated excellent performance in this perspective [2]. More specifically, the basic formulation of k-t BLAST is given by

$$\rho = \bar{\rho} + \Theta F^H \left(F\Theta F^H + \lambda I\right)^{-1} (v - F\bar{\rho}) . \qquad (1)$$

where λ is a regularization parameter, v implies measurement data on k-t space, and Θ denotes the diagonal approximation of signal covariance matrix obtained by just taking low frequency data along k axis at each time and Fourier transforming them to y-f space. In Eq. (1), F corresponds to 2-D Fourier transform from y-f space to k-t space, and the k-t space sampling trajectory depends on whether it is cartesian or radial acquisition. In Eq. (1), $\bar{\rho}$ implies temporal DC term of y-f support. This term is calculated separately by averaging measurement data. Usually, DC signal is dominantly strong compared to other frequencies. Therefore, to reconstruct high frequency signal without interference from the strong DC signal, k-t BLAST and k-t SENSE process DC term separately.

3. DERIVATION OF K-T FOCUSS

FOCUSS is an algorithm originally designed to obtain a sparse solution to the under-determined linear inverse problem.

$$v = F\rho. \qquad (2)$$

In this paper, v corresponds to measurement on k-t space, ρ is the unknown sparse support on x-f space, and F implies 2-D Fourier transform, respectively. Now, FOCUSS considers the following reweighted norm optimization problem [5]:

$$Find : \rho = Wq \qquad (3)$$

where W is a diagonal form of weighting matrix, and q is a solution of the following constrained minimization problem.:

$$\min ||q||_2, \quad \text{subject to } ||v - FWq||_2 < \epsilon \qquad (4)$$

where $|| \cdot ||_2$ denotes the L_2 norm. To solve this problem, FOCUSS obtains the initial estimate of W. In cartesian trajectory, we employ the random sampling pattern with more samples around low frequency region to obtain the initial estimate. In radial trajectory, the initial estimate can be calculated from the over-sampled k-space center region. The constrained optimization problem Eq. (4) can be converted into the un-constrained optimization problem using Lagrangian multiplier, providing a cost function,

$$C(q) = ||v - FWq||_2^2 + \lambda ||q||_2^2 \qquad (5)$$

In a slightly different formulation, ρ is initialized with non-zero values corresponding to DC component of y-f space as done in k-t BLAST/SENSE. In this case, the cost function Eq. (5) can be modified into the following form:

$$C(q) = ||v - F\bar{\rho} - FWq||_2^2 + \lambda ||q||_2^2 \quad \text{where } \rho = \bar{\rho} + Wq. \qquad (6)$$

Then, the optimal solution is given by

$$\rho = \bar{\rho} + \Theta F^H \left(F\Theta F^H + \lambda I\right)^{-1} (v - F\bar{\rho}) \quad \text{where } \Theta = WW^H. \qquad (7)$$

If the optimal solution is found for a given W, then W is updated by taking p power of the previous estimate ρ_n:.

$$W_{n+1} = \begin{pmatrix} |\rho_n(1)|^p & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |\rho_n(N)|^p \end{pmatrix}, \ 1/2 \le p \le 1 . \qquad (8)$$

where $\rho_n(i)$ denotes the i-th elements of the vector ρ_n. Through several iterations, the solution converges. Additionally, by setting p to 0.5, we can easily verify that our k-t FOCUSS asymptotically solves the L_1 minimization problem. Here, L_1 is defined as the sum of the absolute values of the whole data. Specifically, Eq. (4) can be restated as

$$\min ||W_n^{-1}\rho||_2, \quad \text{subject to } ||v - F\rho||_2 < \epsilon \qquad (9)$$

if $p = 0.5$

$$\begin{aligned} ||W_n^{-1}\rho||_2^2 &= \rho^H W_n^{-H} W_n^{-1} \rho \\ &= \rho^H diag(|\rho_{n-1}|^{-1})\rho \\ &\sim \sum_{i=1}^{N} |\rho_{n-1}(i)| \quad \text{as } n \to \infty \\ &= ||\rho||_1 \end{aligned}$$

Therefore, our k-t FOCUSS algorithm is asymptotically optimal from compressed sensing perspective since L_1 minimization is preferred optimization method for compressed sensing. If we compare Eq. (7) with Eq. (1), we can easily figure out that k-t BLAST/SENSE is indeed the first iteration of our k-t FOCUSS algorithm except some detail parameter like p value. In k-t BLAST/SENSE, $p = 1$ and the iteration number is just one. That means k-t BLAST/SENSE is not optimal sparse reconstruction solver compared to our k-t FOCUSS from compressed sensing perspective.

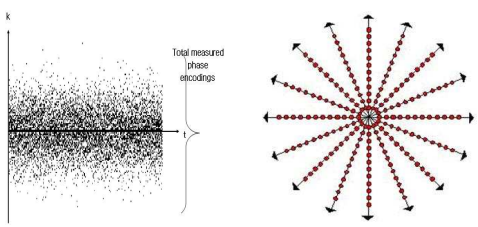
When multiple coils are used, we apply k-t FOCUSS algorithm separately for each coil. After that, the final reconstruction result can be calculated using the least square:

$$\rho = \left(\sum_{i=1}^{N_c} S_i S_i^H \right)^{-1} \left(\sum_{i=1}^{N_c} S_i^H \rho_i \right) \tag{10}$$

wher $S_i, i = 1, \cdots, N_c$ denote the coil sensitivities.

4. IMPLEMENTATION

Based on compressed sensing theory, the data acquisition pattern should be *mutually incoherent*[3, 4]. Also by taking more samples around low frequency region on k-t space, it becomes possible to estimate initial low resolution spectral support without training data set.



(a) (b)

Fig. 1. k-t sampling pattern used in simulation. (a) shows 4x acceleration random sampling pattern for Cartesian. (b) corresponds to the k- space sampling pattern for radial trajectory.

For the case of cartesian trajectory, using the sampling pattern in Figure 1(a), we can directly obtain initial low resolution estimate of spectral support by zero-padded 2-D Fourier transform. For the radial trajectory, at each time frame, we takes 36 regularly under-sampled radon views as shown in Figure 1(b). Then, filtered-backprojection provides a good initial estimation from the over-sampled k-space center region. If the low resolution estimate of y-f support are obtained, we are ready to apply k-t FOCUSS algorithm to reconstruct high resolution dynamic images. The overall flow chart of our k-t FOCUSS algorithm is illustrated in Fig. 2.

5. SIMULATION RESULTS

To validate our algorithm, we have conducted simulation study using the fully sampled cardiac data set. We have acquired 25

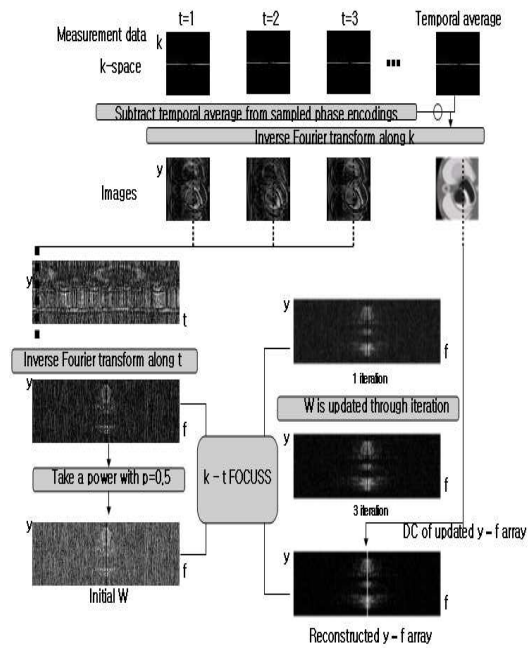


Fig. 2. k-t FOCUSS reconstruction flow chart.

frames of full k-space cardiac sequence of a patient using 1.5 T Philips scanner. The field of view (FOV) was 360.00mm, and the matrix size for scanning was 192 x 192 and the images are reconstructed using 256 x 256 array. The slice thickness was 10.0 mm, and the acquisition sequence was the balanced steady-state free precession (SSFP) with flip angle of 50 degree. The heart frequency was 60 bpm, and retrospective cardiac gating was used. This cardiac cine images are used as ground true reference images to check the reconstruction quality of k-t FOCUSS. We have tested k-t FOCUSS algorithm for several down sampling ratio. The reconstruction quality gets better as iteration increases and the motion artifact and spatial aliasing artifact are effectively removed even under 8x down sampling rate. Figure 3 shows the reconstruction results for 4x down sampling rate in cartesian random sampling cases. We can figure out that after 5 iterations the result gets sharper and detail structures like valves and vessels are clearly seen compared to after 1iteration. Also, by using multiple coils, we could improve the reconstruction quality even under severe down sampling rate like 8x, 16x acceleration.

We have also applied our k-t FOCUSS to radial trajectory as shown in Figure 4. At each time frame, 36 under-sampled views are taken and the segment order is shifted at each time frame. We can clearly see that the streak artifacts are completely removed with more iteration and accurate cardiac four chamber views are obtained.

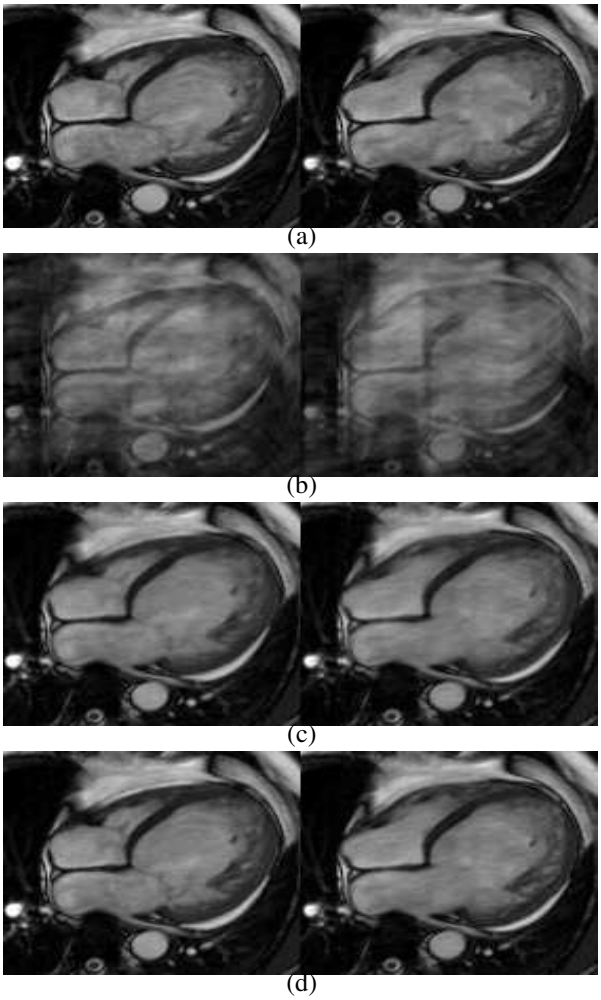


Fig. 3. k-t FOCUSS results for cartesian sampling: (a) original images, (b) zero-padded Fourier transform from measurement data, (c) after 1 iteration, (d) after 5 iterations. The left and right column show the different frames

6. CONCLUSION

We have demonstrated that our k-t FOCUSS is asymptotically optimal from the compressed sensing theory point of view. The k-t FOCUSS was successfully applied for high resolution reconstruction of cardiac MR image even under severely limited samples where the aliasing free high resolution reconstruction was not possible using the conventional k-t BLAST and k-t SENSE. Furthermore, we have demonstrated that k-t BLAST and k-t SENSE are the special case of k-t FOCUSS. More exactly, k-t BLAST and k-t SENSE turn out the first iteration result of k-t FOCUSS. From the point of view for the compressed sensing theory, there are still many factors to be modified and to be tried. We expect that this paper will open this new area of research.

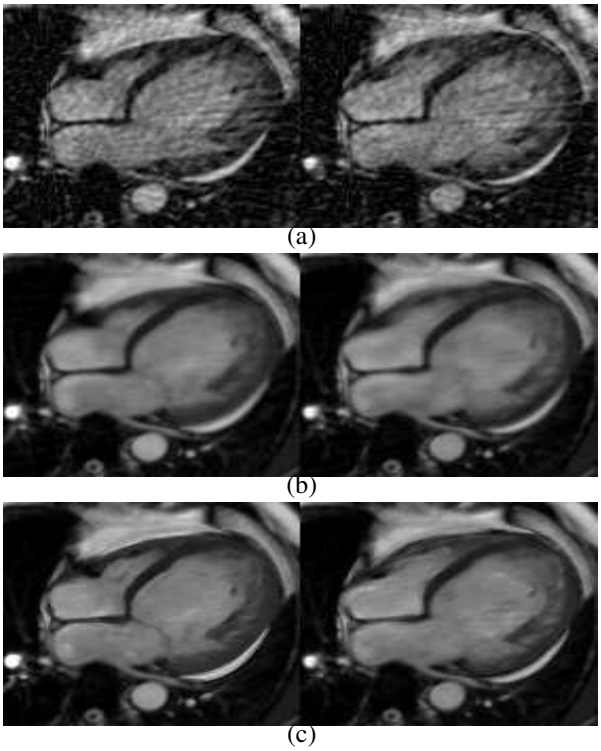


Fig. 4. k-t FOCUSS results for radial sampling: (a) direct filtered back projection from angularly undersampled data, (b) after one iteration, and (d) after three iterations, respectively. The left and right column show the different frames

7. REFERENCES

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