A NOVEL K-SPACE ANNIHILATING FILTER METHOD FOR UNIFICATION BETWEEN COMPRESSED SENSING AND PARALLEL MRI

Kyong Hwan Jin, Dongwook Lee, Jong Chul Ye

Bio-Imaging and Signal Processing Lab., Dept. Bio & Brain Engineering
Korea Advanced Institute of Science & Technology (KAIST)

ABSTRACT

In this paper, we propose a novel k-space method called ALOHA (Annihilating filter based LOw-rank Hankel matrix Approach) that unifies parallel imaging and compressed sensing as a k-space data interpolation problem. Specifically, ALOHA employs annihilating filter relationships originated from the intrinsic image property originated from the finite rate of innovation model, as well as the multi-coil acquisition physics. By interchanging the annihilating filter with the k-space measurement, a rank-deficient block Hankel structured matrix can be obtained, whose missing elements can be restored by a low rank matrix completion algorithm. To exploit the low rank Hankel structure, we develop an alternating direction method of multiplier (ADMM) method with initialisation from low rank matrix fitting (LMaFit) algorithm. Additionally, we develop a novel pyramidal representation of the Hankel structured matrix to reduce the computational complexity of the algorithm. ALOHA can be universally applied to compressed sensing MRI as well as parallel imaging for both static and dynamic applications. Experimental results with real in vivo data confirmed that ALOHA outperforms the existing state-of-the-art parallel and compressed sensing MRI.

Index Terms— Parallel MRI, Hankel matrix, annihilation filter, finite rate of innovation

1. INTRODUCTION

MRI is an imaging system that sequentially acquires k-space data corresponding to the Fourier transform of an underlying image. This enables us to apply various advanced signal processing techniques. Recently, the compressed sensing theory has been studied in accelerated MRI. Compressed sensing can recover original signals from reduced samples by exploiting sparsity of target signals in specific domain, and incoherent sensing scheme using irregular sampling such as Gaussian random or Poisson disc are necessary for improved reconstruction quality. Accurate MRI reconstruction from reduced data without significant distortion makes the compressed sensing a hot research topic with many applications such as dynamic cardiac MRI[1].

On the other hand, parallel MRI (pMRI) utilizes the diversity of coil sensitivity maps that are multiplied with an underlying image. The diversity in coil sensitivites provides additional spatial information for the underlying signal, resulting in accelerated reconstruction. The representative parallel imaging algorithms such as SENSE (sensitivity encoding) [2] or GRAPPA [3] demand the regularly sampled k-space data for computationally efficient reconstruction.

As the two approaches all aim at accelerated acquisition with reduced k-space data, we can easily expect many research activities to synergistically combine parallel imaging and compressed sensing for further acceleration in MR acquisition. The multichannel version of k-t FOCUSS [1] is one of the typical examples of such approaches. On the other hand, l_1-SPIRiT (l_1- Self-consistent Parallel Imaging ReconstrucTion) utilizes the GRAPPA type constraint as an additional regularisation term for compressed sensing problem. In both approaches, the accurate estimation of the coil sensitivity maps or GRAPPA kernel is important to fully utilise the diversities in coil sensitivity. However, the conflicting requirements of *incoherent* and *regular* sampling often prohibits the full utilisation of the compressed sensing and parallel imaging in their maximum capabilities, and the current results from the combination are often disappointing.

In this paper, we propose a unified k-space reconstruction framework called ALOHA (Annihilating filter based LOw-rank Hankel matrix Approach) that overcomes the aforementioned limitations to achieve the maximum synergistic combination of the parallel imaging and compressed sensing.

2. ANNIHILATION PROPERTIES IN MRI

In this paper, we introduce novel annihilating filter relationships. The first kind of annihilating filter comes from the intrinsic image property. Specifically, if the underlying signal $x(t)$ comes from the finite rate of innovation [4] or it has finite support, we can show that there exists function $h(t)$ such that $h(t)x(t) = 0$. Next, in pMRI, the unknown image $y_i(t)$ that we can reconstruct from the i-th k-space measurement can be

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represented as

$$y_i(t) = s_i(t)x(t), \quad i = 1, \cdots, C, \tag{1}$$

where $s_i(t)$ denotes the i-th coil sensitivity maps and $x(t)$ is underlying unknown image. Then, we have the following equality:

$$s_j(t)y_i(t) = s_i(t)y_j(t), \quad i, j = 1, \cdots, C, \text{ and } i \neq j. \tag{2}$$

If the underlying signals can be modeled as non-uniform splines with order $R-1$ and the coil sensitivity varies more smoothly than the underlying signals, the annihilating properties in (1) and (2) hold for the R-th derivative of the signals, i.e. $y_i^R(t) = d^R y_i(t)/dt^R$. Accordingly, the corresponding k-space measurement should satisfy the following convolution equality:

$$(H_i * Y_i^R)[n] = 0, \quad i = 1, \cdots, C, \tag{3}$$

$$(S_j * Y_i^R)[n] = (S_i * Y_j^R)[n], \quad i, j = 1, \cdots, C, \text{ and } i \neq j, \tag{4}$$

where H_i, S_i and Y_i^R denote the Fourier series coefficients of $h_i(t) = h(t)s_i(t)$, $s_i(t)$ and $y_i^R(t)$, respectively. By extending 1D equations, we can find a 2-D annihilating filter $H[m, n]$ such that

$$(H * Y^R)[m, n] = \sum_i \sum_j H[m-i, n-j] Y^R[i, j] = 0 \tag{5}$$

where $R = 0, 1, \cdots$ and

$$Y^R[m, n] = \frac{1}{2}\left(\left(\frac{i2\pi m}{\tau}\right)^R + \left(\frac{i2\pi n}{\tau}\right)^R\right) Y[m, n], \tag{6}$$

where (6) comes from the relationship between R-th derivative and the corresponding Fourier series coefficients. This model is useful since images are not usually sparse but the finite difference operation makes images sparse. Especially, when $R = 2$, $Y^R[m, n]$ is identical to k-space data from the Laplacian of the image, $\nabla^2 y$. For parallel imaging, Laplacian weighting can be applied to (4).

Let the (weighted) k-space data in the region of interest be denoted by $\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 & \cdots & \mathbf{y}_N \end{bmatrix} \in \mathbf{C}^{M \times N}$, and the corresponding annihilation filter, $\mathbf{H} = \begin{bmatrix} \mathbf{h}_{-n} & \cdots & \mathbf{h}_n \end{bmatrix} \in \mathbf{C}^{(2m+1)\times(2n+1)}$, then, the annihilating filter can be represented as

$$\mathscr{C}\{\mathbf{H}\}\textsc{Vec}(\mathbf{Y}) = \mathbf{0} \tag{7}$$

where $\mathscr{C}\{\mathbf{H}\}$ denotes the 2-D chopped convolution matrix.

By interchanging the annihilating filter and the signal in (3) and (4) and representing them using convolution matrix, we have

$$\mathscr{H}\{\mathbf{Y}_i\}\mathfrak{h}_i = \mathbf{0}, \quad \mathfrak{h}_i \triangleq \textsc{Vec}(\mathbf{H}_i)^\vee, \quad i = 1, \cdots, C, \tag{8}$$

$$\mathscr{H}\{\mathbf{Y}_i\}\mathfrak{h}_j - \mathscr{H}\{\mathbf{Y}_j\}\mathfrak{h}_i = \mathbf{0}, \quad i, j = 1, \cdots, C, \quad i \neq j \tag{9}$$

where $\vee$ denotes the order reversal operator, $\mathscr{H}\{\mathbf{Y}\} \in \mathbb{R}^{(M-2m)(N-2n)\times(2m+1)(2n+1)}$ is the block Hankel structured matrix within ROI constructed as

$$\mathscr{H}\{\mathbf{Y}\} = \begin{bmatrix} \mathcal{H}\{\mathbf{y}_1\} & \mathcal{H}\{\mathbf{y}_2\} & \cdots & \mathcal{H}\{\mathbf{y}_{2n+1}\} \\ \mathcal{H}\{\mathbf{y}_2\} & \mathcal{H}\{\mathbf{y}_3\} & \cdots & \mathcal{H}\{\mathbf{y}_{2n+2}\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{H}\{\mathbf{y}_{N-2n}\} & \mathcal{H}\{\mathbf{y}_{N-2n+1}\} & \cdots & \mathcal{H}\{\mathbf{y}_N\} \end{bmatrix},$$

where $\mathcal{H}\{\mathbf{y}_i\}$ is a Hankel matrix consisted with $\mathbf{y}_i$.

For parallel MRI, we can construct the augmented k-space data matrix to impose two annihilation properties, at the same time,

$$\boldsymbol{\mathcal{Y}} = \begin{bmatrix} \mathscr{H}\{\mathbf{Y}_1\} & \mathscr{H}\{\mathbf{Y}_2\} & \cdots & \mathscr{H}\{\mathbf{Y}_C\} \end{bmatrix}. \tag{10}$$

Then, (8) and (9) can be represented by

$$\boldsymbol{\mathcal{Y}}\mathbf{Q} = 0, \quad \boldsymbol{\mathcal{Y}}\mathbf{B} = 0$$

where $\mathbf{Q}$ and $\mathbf{B}$ come from the single coil and multi-coil annihilating filters, respectively. Even though above two annihilation properties are derived for the k-space samples in k_x-k_y-coil domain, it is straightforward to extend this to k-t-coil domain owing to the sparsity in y-f domain. Only difference between k_x-k_y and k-t, is no k-space weighting on k-t-coil domain derived from (6) because k-space weighting is not helpful for making y-f signals be more sparse.

3. LOW RANK HANKEL MATRIX COMPLETION ALGORITHM

The annihilation structure implies that there exists null space in $\boldsymbol{\mathcal{Y}}$. This relationship leads us to an algorithm that estimates the missing elements in $\boldsymbol{\mathcal{Y}}$ using a low rank structured matrix completion problem. For solving the low rank matrix completion problem, we employ the SVD-free structured rank minimization algorithm [5] with an initialization using the low-rank factorization model (LMaFit) algorithm [6]. Specifically, the algorithm is based on the following observation :

$$\|\mathbf{A}\|_* = \min_{\mathbf{U},\mathbf{V}:\mathbf{A}=\mathbf{U}\mathbf{V}^H} \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 \tag{11}$$

Here $\|\mathbf{A}\|_*$ denotes the nuclear norm of the matrix. Accordingly, we are interested in the nuclear norm minimization under the matrix factorization constraint:

$$\begin{aligned} \min_{\boldsymbol{\mathcal{Y}}} \quad & \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 \\ \text{subject to} \quad & \boldsymbol{\mathcal{Y}} = \mathbf{U}\mathbf{V}^H \\ & \mathbf{Y}_i[p, q] = M_i[p, q], \ \forall (p, q) \in \Omega, \end{aligned}$$

where $M_i[p, q]$ denote the i-th coil measured k-space sampling on domain Ω. By combining the two constraints, we have the following cost function for ADMM:

$$L(\mathbf{U},\mathbf{V},\boldsymbol{\mathcal{Y}},\boldsymbol{\Lambda}) := \iota_S(\boldsymbol{\mathcal{Y}}) + \frac{1}{2}\left(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2\right) + \frac{\mu}{2}\|\boldsymbol{\mathcal{Y}} - \mathbf{U}\mathbf{V}^H + \boldsymbol{\Lambda}\|_F^2 \quad (12)$$

where $\iota_S(\boldsymbol{\mathcal{Y}})$ denotes the indicator function that represents the k-space measured data consistency. This ADMM problem can be iteratively solved with initializations from LMaFit[6] with pyramidal representation of k-space data as shown in Fig.1. The pyramidal representation enables us to save computation time for matrix completion resulting from reduced size of patches. Furthermore, different number of iterations can be used at each level of pyramid to exploit the signal distribution and to save the computational complexity. In case of k_x-k_y-coil domain, current pyramidal patches are obtained from the previous level as a half size of patch including DC as a center point. For the case of k-t measurement, pyramidal patches are obtained from the previous level as a half size of the patch along the phase encoding direction only including DCs as a line.

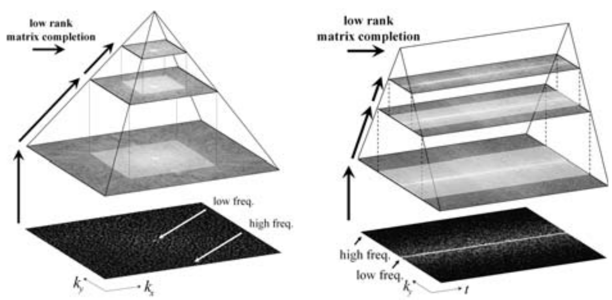

Fig. 1. Pyramidal decomposition of k_x-k_y domain(left), and k-t domain(right).

4. RESULTS

To assess the performance of ALOHA for single coil compressed sensing imaging, a brain k-space data was obtained with Siemens Verio 3T scanner using 2D SE sequence. The acquisition parameters were TR/TE $= 4000/100$ms, 256×512 acquisition matrix size, 6 z-slice with 5mm slice thickness and FOV was 240×240mm^2, the number of coils was four. A retrospective down-sampling mask was generated by two dimensional Gaussian distribution including the central 7×7 region. For the compressed sensing approach, we used split Bregman method constrained by total varitaion (TV) [7] with same data and sampling masks. To evaluate results quantitatively, we use two metrics; NMSE and SSIM index [8].

In single coil experiment (Fig. 2), ALOHA showed better performance in both quantitative values and visual quality. To evaluate the performance of ALOHA in static parallel imaging (Fig. 3), the same brain data in Fig. 2 was used. At the reduction factor of five, images was reconstructed using measurements from 4 coils. The ALOHA reconstruction experiments were conducted under a two dimensional Gaussian

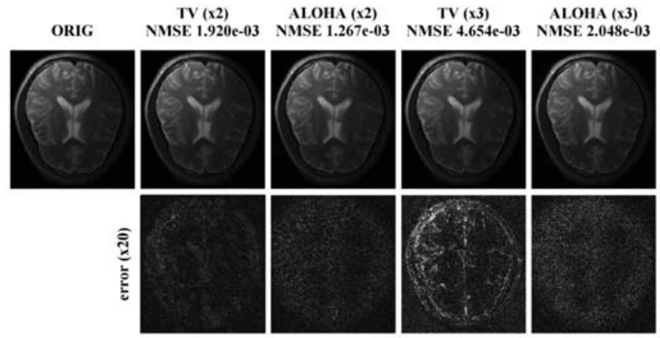

Fig. 2. Reconstructions from split Bregman method with total variation [7] and ALOHA under multi-fold down sampling patterns from a single coil measurement.

sampling including the central 7×7 region. As for existing methods, we used the identical data and sampling masks for GRAPPA [3], SAKE [9], and SAKE with ESPIRiT [10]. Note that GRAPPA requires ACS lines, so the effective down sampling ratio was 4.61 with the number of ACS lines of 10. SAKE and SAKE with ESPIRiT are basically low rank matrix completion algorithm for the matrix collected from the whole k-space data. However, SAKE with ESPIRiT reduces the computational burden of the original SAKE by performing low rank matrix completion only for 65×65 central regions, after which GRAPPA kernels are estimated using the reconstruction data. The estimated GRAPPA kernel are used to estimate the remaining k-space missing data. The parameters for SAKE and SAKE with ESPIRiT were chosen such that they provided the best reconstruction results. The NMSE values for reconstruction results by GRAPPA, SAKE, SAKE with ESPIRiT, and ALOHA in Fig. 3 showed that ALOHA was most effective. In the second row of Fig. 3, we observed that proposed method outperformed other compared algorithms.

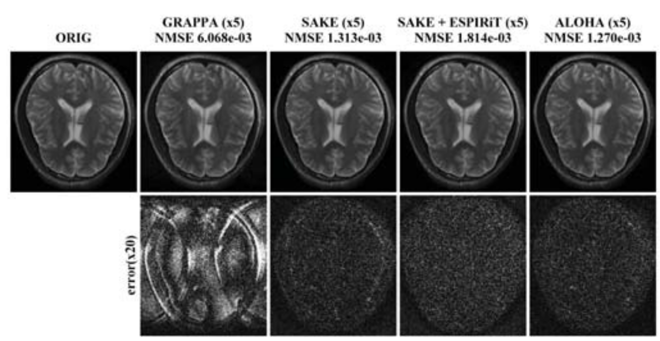

Fig. 3. Reconstructions results by GRAPPA [3], SAKE [9], SAKE with ESPIRiT [10] and the ALOHA method using five fold downsampled data.

For dynamic imaging applications, a cardiac cine data set was acquired using a 3T whole-body MRI scanner (Siemens;

Tim Trio). The acquisition sequence was bSSFP and prospective cardiac gating was used. The imaging parameters include: FOV$=\ 300 \times 300\text{mm}^2$, acquisition matrix size$=$ 128×128, TE/TR $=\ 1.37/2.7$ms, receiver bandwidth $=\ 1184$Hz/pixel, and flip angle $=\ 40°$. The number of cardiac phases is 23 and the temporal resolution was 43.2ms. The number of coils used in the reconstruction was four. As for comparision, k-t FOCUSS [1] was used. For best reconstruction in k-t FOCUSS, separate regularization parameters are used for each coil. After the reconstructions on each coil independently, SSoS image was calculated across 4 coils. The NMSE values for two algorithms, k-t FOCUSS and ALOHA at reduction factor of six using Gaussian sampling were 5.66×10^{-3} and 2.78×10^{-3}, respectively. In case of eigth fold acceleration, the resulting NMSEs are 8.1×10^{-3} and 3.905×10^{-3}, respectively.

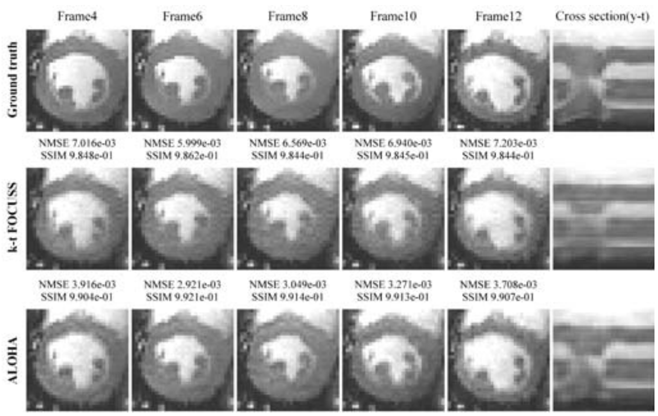

Fig. 4. Magnified view of reconstructed parallel dynamic images at the 6x acceleration factor acquired from four coils.

5. CONCLUSION

We proposed a novel k-space method called ALOHA that unifies the parallel imaging and compressed sensing, as a k-space data interpolation problem. ALOHA successfully employed annihilating filter relationships originated from the intrinsic image property as well as the multi-coil acquisition physics. These annihilation filters imposed the intrinsic image property from the R-th derivative of k-space data based on the finite rate of innovation. Rank-deficient block Hankel structured matrix was recovered by a structured low rank matrix completion algorithm using ADMM method with LMaFit combined with a novel pyramidal representation of the Hankel structured matrix. We demonstrated the reconstruction results from ALOHA outperformed other existing algorithms in both static parallel and dynamic parallel MRI. We believe that ALOHA can be a general solution of compressed sensing MRI as well as parallel imaging for both static and dynamic applications.

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