PATCH BASED LOW RANK STRUCTURED MATRIX COMPLETION FOR ACCELERATED SCANNING MICROSCOPY

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ABSTRACT

We propose a low rank structured matrix completion algorithm for image inpainting problems originated from scanning microscopy. The proposed method exploits the annihilation property observed in Gaussian Markov Random Field (GMRF) or partial differential equation (PDE)-based inpainting approaches. By utilizing the commutative property of the convolution, the annihilation property is embodied into rank-deficient block Hankel structure data matrices and the image inpainting problem is converted into low-rank structured matrix completion problem. To solve the structured low-rank matrix completion problem, an alternating direction method of multiplier (ADMM) method is used with factorization matrix initialization using the low rank matrix fitting (LMaFit) algorithm. Experimental results showed that the proposed method outperforms the existing state-of-the-art image inpainting methods.

Index Terms— Scanning microscopy, structured block Hankel matrix, low rank matrix completion, ADMM, LMaFit

1. INTRODUCTION

An inpainting technique is a method to fill in missing pixels in an image by exploiting redundant information from adjacent pixels. Applications of inpainting can be found in many scientific and engineering contexts: for example, in scanning microscopy, by reducing the scanning positions, the acquisition time can be proportionally reduced. However, the missing pixels that were not scanned need to be interpolated appropriately.

Important recent research trend in inpainting comes from data driven approaches. For example, the kernel regression method [1] has been developed to adapt the local nature of the images using data-driven kernels. Meanwhile, sparsity driven interpolation methods have been extensively studied. Among these studies, K-SVD [2] is to find the data-adaptive local basis and fit data based on the sparsity of the regression coefficients. Another important class of classical inpainting methods is based on the statistical and deterministic structure of the images. For example, Gaussian Markov random field (GMRF) model is often used to model the textures of images in a stochastic sense, which is again used to estimate the missing data [3]. A deterministic generalisation has been also extensively studied using variational framework and partial differential equation (PDE) for diffusion process. The approaches in the most of aforementioned inpainting methods are to find optimized interpolation filters or kernels that enable the estimation of missing pixels.

Different from those approaches, one of the main contributions of the proposed method is to show that the estimation step of a kernel or an interpolation filter is not necessary if the interpolation filter is *locally* spatially invariant. Specifically, we reveal that the fixed point solution of many inpainting algorithms such as GMRF or anisotropic diffusion satisfies an interesting annihilation property at each image patch. Specifically, thanks to the commutative property of convolution, block Hankel structured data matrix can be obtained from each image patch. Accordingly, assuming that an image patch lives in a low dimensional manifold, the problem is reduced to a low rank matrix completion problem for Hankel structured matrix [4, 5], from which the missing samples in the patch can be easily obtained. Such interpolation steps are applied on overlapping patches to obtain the missing pixel values and the estimated values are averaged to avoid any blocking artifact.

The proposed approach exploiting the low rankness in block Hankel structructured matrix is shown effective in scanning microscopy such as STED microscopy and confocal microscopy which have to scan a sample point by point. By demonstrating superior image inpainting results from such measurement, our result section confirms the feasibility of proof-of-concept system.

2. RPOBLEM FORMULATION

The annihilation property in GMRF and anisotropic diffusion can be exploited for image inpainting. More specifically, at fixed point solution, the ML solution for GMRF based inpainting should satisfy

$$\mathbf{B}\mathbf{x} = 0$$

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where $\mathbf{B}$ is the matrix representation of GMRF non-causal filter. The locally shift invariant property can be represented as convolution operator which has block Toeplitz structure in descrete domain. Therefore, we are interested in partitioning images into overlapping patches, within which the same annihilation property holds.

In this case, if we let $\mathbf{X} \in \mathbb{R}^{M\times N}$ be an image patch, then we can find a 2-D interpolation filter $\mathbf{H} = [\mathbf{h}_{-n}, \cdots, \mathbf{h}_n] \in \mathbb{R}^{(2m+1)\times(2n+1)}$ such that the annihilation property holds by representing $\mathbf{B}$ as the convolution matrix for $\mathbf{H}$. Within a patch, the annihilation relation should hold only inside of the patch as shown in Fig. 1 where the matrix $\mathbf{B}$ is defined as:

$$\mathbf{B} = \mathscr{C}\{\mathbf{H}\} \tag{1}$$

where $\mathscr{C}\{\mathbf{H}\}$ denotes the 2-D chopped convolution matrix:

$$\mathscr{C}\{\mathbf{H}\} = \begin{bmatrix} \mathcal{C}\{\mathbf{h}_n\} & \cdots & \mathcal{C}\{\mathbf{h}_{-n}\} & & & \mathbf{0} \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ \mathbf{0} & & & \mathcal{C}\{\mathbf{h}_n\} & \cdots & \mathcal{C}\{\mathbf{h}_{-n}\} \end{bmatrix} \in \mathbb{R}^{(M-2m)(N-2n)\times MN}. \tag{2}$$

where a chopped 1-D convolution matrix $\mathcal{C}\{\mathbf{h}_i\}$ with the filter $\mathbf{h}_i \in \mathbb{R}^{2m+1}$ is defined by

$$\mathcal{C}\{\mathbf{h}_i\} = \begin{bmatrix} \mathbf{h}_i(m) & \cdots & \mathbf{h}_i(-m) & & & 0 \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ 0 & & & \mathbf{h}_i(m) & \cdots & \mathbf{h}_i(-m) \end{bmatrix} \in \mathbb{R}^{(M-2m)\times M}. \tag{3}$$

Now we have the following commutative property :

$$\mathcal{C}\{\mathbf{h}_j\}\mathbf{x}_i = \mathcal{H}\{\mathbf{x}_i\}\mathbf{h}_j^{\vee}, \tag{4}$$

where the superscript $^{\vee}$ denotes the order reversal operator and $\mathcal{H}\{\mathbf{x}_i\}$ is a Hankel matrix given by

$$\mathcal{H}\{\mathbf{x}_i\} = \begin{bmatrix} \mathbf{x}_i(1) & \mathbf{x}_i(2) & \cdots & \mathbf{x}_i(2m+1) \\ \mathbf{x}_i(2) & \mathbf{x}_i(3) & \cdots & \mathbf{x}_i(2m+2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_i(M-2m) & \mathbf{x}_i(M-2m+1) & \cdots & \mathbf{x}_i(M) \end{bmatrix} \in \mathbb{R}^{(M-2m)\times(2m+1)}. \tag{5}$$

Using Eq. (4), the annihilation property can be represented as

$$\mathscr{C}\{\mathbf{H}\}\textsc{vec}(\mathbf{X}) = \mathscr{H}\{\mathbf{X}\}\textsc{vec}(\mathbf{H})^{\vee} = \mathbf{0}, \tag{6}$$

where patch-based block Hankel structured matrix $\mathscr{H}\{\mathbf{X}\}$ is constructed as

$$\mathscr{H}\{\mathbf{X}\} = \begin{bmatrix} \mathcal{H}\{\mathbf{x}_1\} & \mathcal{H}\{\mathbf{x}_2\} & \cdots & \mathcal{H}\{\mathbf{x}_{2n+1}\} \\ \mathcal{H}\{\mathbf{x}_2\} & \mathcal{H}\{\mathbf{x}_3\} & \cdots & \mathcal{H}\{\mathbf{x}_{2n+2}\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{H}\{\mathbf{x}_{N-2n}\} & \mathcal{H}\{\mathbf{x}_{N-2n+1}\} & \cdots & \mathcal{H}\{\mathbf{x}_N\} \end{bmatrix} \in \mathbb{R}^{(M-2m)(N-2n)\times(2m+1)(2n+1)}. \tag{7}$$

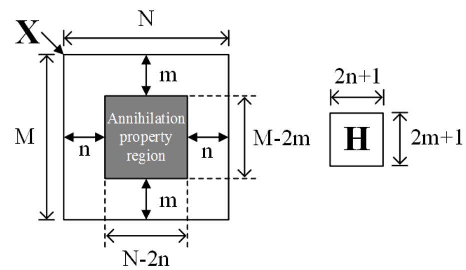

Fig. 1. Region within a patch where the annihilation property holds.

3. METHODS

The annihilation structure in (6) implies that there exists null space in $\mathscr{H}\{\mathbf{X}\}$. This means that the block Hankel matrix is rank deficient. This relationship leads us to an algorithm that estimates the missing elements in $\mathscr{H}\{\mathbf{X}\}$. More specifically, the missing element estimation problem becomes a low rank structured matrix completion problem:

$$\begin{aligned} \min \quad & \textsc{Rank}(\mathscr{H}\{\mathbf{X}\}) && (8) \\ \text{subject to} \quad & \mathbf{X}(i,j) = \mathbf{M}(i,j), \quad \forall (i,j) \in \Omega, && (9) \end{aligned}$$

where $\mathbf{M}$ is the measurement matrix with measured values at positions $(i,j) \in \Omega$ and 0's elsewhere. We employ the SVD-free structured rank minimization algorithm [4] with an initialization using the low-rank factorization model (LMaFit) algorithm [5]. Specifically, the algorithm is based on the following observation [6]:

$$\|\mathbf{A}\|_* = \min_{\mathbf{U},\mathbf{V}:\mathbf{A}=\mathbf{U}\mathbf{V}^H} \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 \tag{10}$$

Here $\|\mathbf{A}\|_*$ denotes the nuclear norm of the matrix $\mathbf{A}$. Accordingly, we are interested in the nuclear norm minimization under the matrix factorization constraint:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 && \\ \text{subject to} \quad & \mathscr{H}\{\mathbf{X}\} = \mathbf{U}\mathbf{V}^H && (11) \\ & \mathbf{X}(i,j) = m_{ij}, \quad \forall (i,j) \in \Omega, && (12) \end{aligned}$$

The first constraint (11) can be handled using alternating direction method of multiplier (ADMM) [7]. The second constraint (12) is a set constraint imposing consistency of $\mathbf{X}$.

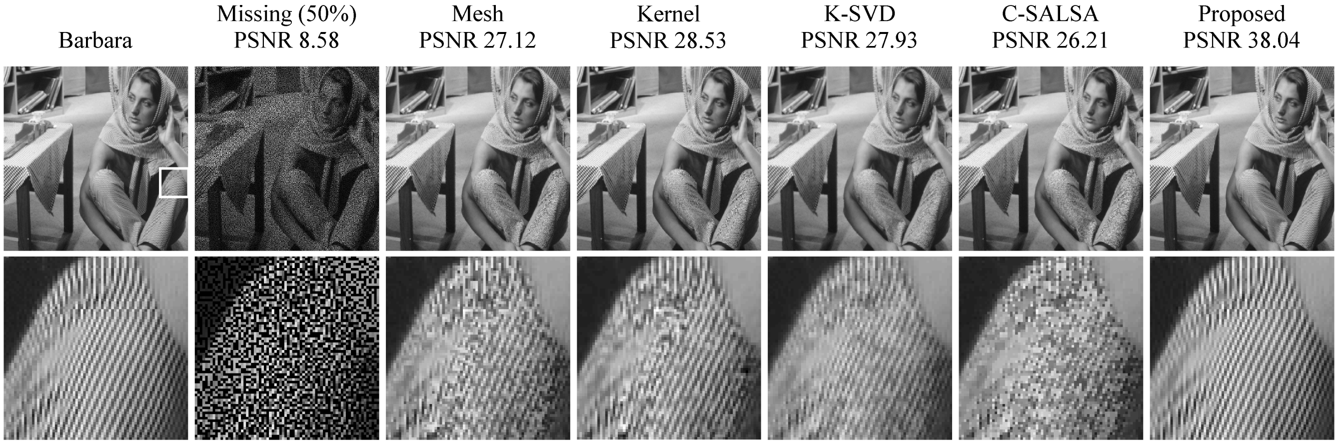

Fig. 2. Reconstructed Barbara images using various inpainting algorithms.

Therefore, by combining the two constraints, we have the following cost function for ADMM:

$$L(\mathbf{U},\mathbf{V},\mathbf{X},\mathbf{\Lambda}) := \iota_C(\mathbf{X}) + \frac{1}{2}\left(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2\right) + \frac{\mu}{2}\|\mathscr{H}\{\mathbf{X}\} - \mathbf{U}\mathbf{V}^H + \mathbf{\Lambda}\|_F^2 \quad (13)$$

where $\iota_C(\mathbf{X})$ denotes the indicator function. One of the advantages of the ADMM formulation is that each subprogram is simply obtained from (13). More specifically, we have

$$\mathbf{X}^{(k+1)} = \arg\min_{\mathbf{X}} \iota_C(\mathbf{X}) + \frac{\mu}{2}\|\mathscr{H}\{\mathbf{X}\} - \mathbf{U}^{(k)}\mathbf{V}^{(k)H} + \mathbf{\Lambda}^{(k)}\|_F^2 \quad (14)$$

$$\mathbf{U}^{(k+1)} = \arg\min_{\mathbf{U}} \frac{1}{2}\|\mathbf{U}\|_F^2 + \frac{\mu}{2}\|\mathscr{H}\{\mathbf{X}^{(k+1)}\} - \mathbf{U}\mathbf{V}^{(k)H} + \mathbf{\Lambda}^{(k)}\|_F^2 \quad (15)$$

$$\mathbf{V}^{(k+1)} = \arg\min_{\mathbf{V}} \frac{1}{2}\|\mathbf{V}\|_F^2 + \frac{\mu}{2}\|\mathscr{H}\{\mathbf{X}^{(k+1)}\} - \mathbf{U}^{(k+1)}\mathbf{V}^H + \mathbf{\Lambda}^{(k)}\|_F^2 \quad (16)$$

$$\mathbf{\Lambda}^{(k+1)} = \mathscr{H}\{\mathbf{X}^{(k+1)}\} - \mathbf{U}^{(k+1)}\mathbf{V}^{(k+1)H} + \mathbf{\Lambda}^{(k)} \quad (17)$$

It is easy to show that the first step can be simply reduced to

$$\mathbf{X}^{(k+1)} = \mathbf{P}_{\Omega^c}\mathscr{H}^\dagger\left\{\mathbf{U}^{(k)}\mathbf{V}^{(k)H} - \mathbf{\Lambda}^{(k)}\right\} + \mathbf{P}_\Omega\mathbf{M}, \quad (18)$$

where $\mathbf{P}_\Omega$ is a projection mapping on the set Ω and $\mathscr{H}^\dagger$ corresponds to the Penrose-Moore pseduo-inverse mapping from our block Hankel structure to a patch. Next, the subprogram for $\mathbf{U}$ and $\mathbf{V}$ can be easily calculated by taking the derivative with respect to each matrix, and we have

$$\mathbf{U}^{(k+1)} = \mu\left(\mathscr{H}\{\mathbf{X}^{(k+1)}\} + \mathbf{\Lambda}^{(k)}\right)\mathbf{V}^{(k)}\left(I + \mu\mathbf{V}^{(k)H}\mathbf{V}^{(k)}\right)^{-1}, \quad (19)$$

$$\mathbf{V}^{(k+1)} = \mu\left(\mathscr{H}\{\mathbf{X}^{(k+1)}\} + \mathbf{\Lambda}^{(k)}\right)^H\mathbf{U}^{(k+1)}\left(I + \mu\mathbf{U}^{(k+1)H}\mathbf{U}^{(k+1)}\right)^{-1}. \quad (20)$$

Initializations of $\mathbf{U}$ and $\mathbf{V}$ are obtained from an algorithm called low-rank factorization model (LMaFit) [5]. LMaFit solves a linear equation with respect to $\mathbf{U}$ and $\mathbf{V}$ to find their updates and relaxes the updates by taking the average between the previous iteration. Moreover, the rank update can be done automatically. LMaFit uses QR factorization instead of SVD, so it is also computationally efficient. However, the LMaFit alone cannot recover the block Hankel structure, which is the reason we use ADMM later to impose the structure.

4. RESULTS

To validate the algorithm efficacy, we first performed experiments using Barbara image where various textures makes the inpainting difficult. We compared the proposed method with triangular based linear mesh interpolation (Mesh), steering Kernel regression [1], K-SVD [2] and C-SALSA [8]. The data was reduced by uniformly random down-sampling mask by a factor of two. As shown in Fig. 2, the resultant image from the proposed method showed better texture pattern than from the others.

We performed the same inpainting using Lissajous scanning pattern which can be easily implemented in hardware by modulation x- and y-axis of a scanning apparatus separately. Image of endoplasmic reticulum (ER) labeled with tdEos fused to reticulon-4 in U2OS cells was taken on a modified efi-fluorescence microscope (Olympus IX71). The size of view in Fig. 3 is $15.6 \times 15.6\ \mu\text{m}^2$. The second column of Fig. 3 showed the measurement position from Lissajous scanning, which was retrospectively generated from an fully sampled image. We observed that the proposed method outperformed all existing algorithms. As shown in magnified images, the details are smoothed in other approaches, whereas the proposed method preserved the details. The PSNR values also confirm the superiority of our algorithm.

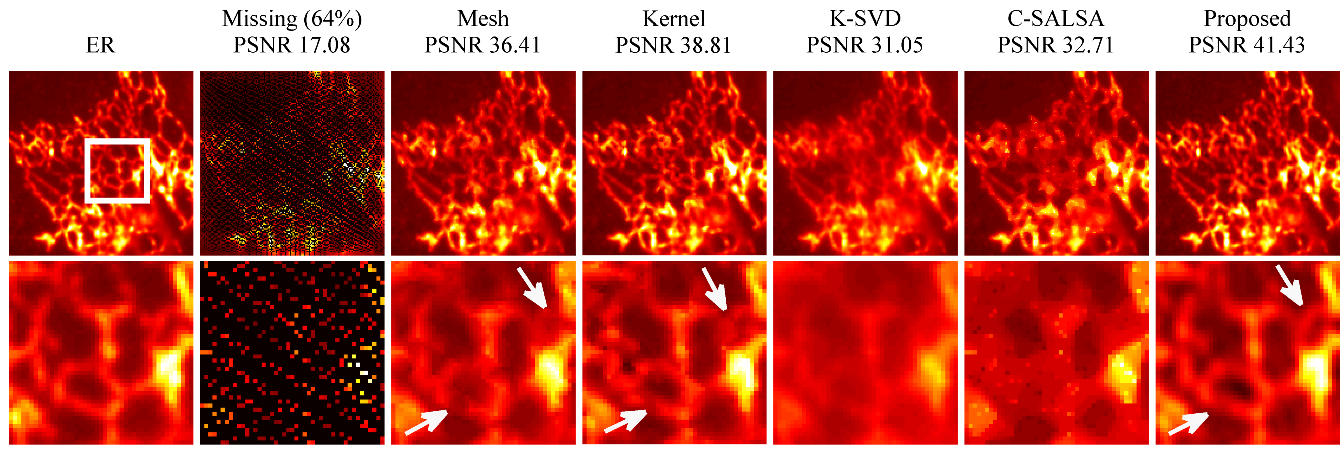

Fig. 3. Reconstructed Endoplasmic Reticulum images using various inpainting algorithms. The sampling patterns was obtained from Lissajous scanning.

5. CONCLUSION

We proposed a novel high performance data inpainting algorithm using low rank matrix completion, which can be used for scanning mircroscopy with irregular scanning pattern. We demonstrated that the proposed method can recover superior reconstruction results, both quantitatively and qulitatively. The main theoretical background of the proposed method comes from annihilating filter relationship originated from GMRF or diffusion equation based inpainting approaches. With regular and simple reconstruction strategy, we believe that our method has significant potential in many inpainting problems.

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