

Improved Temporal Resolution TWIST Reconstruction using Annihilating Filter-based Low-rank Hankel Matrix

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Synopsis

In dynamic contrast enhanced (DCE) MRI, temporal and spatial resolution can be improved by time-resolved angiography with interleaved stochastic trajectories (TWIST). However, due to view sharing, the temporal resolution of TWIST is not a true one. To overcome this limitation, we employ recently proposed annihilating filter-based low rank Hankel matrix approach (ALOHA) that interpolates the missing k-space data by performing low-rank matrix completion of weighted Hankel matrix. In vivo results showed considerably better temporal resolution than standard TWIST reconstruction.

Purpose

TWIST has been widely used in clinic because of its improved temporal resolution¹. However, the periphery of k-space data from several frames should be combined, so the temporal resolution of TWIST is not a true one. TWIST sampling pattern is designed such that the integrated k-space data leads to a uniform undersampling so that it can be reconstructed using GRAPPA². Accordingly, if we just use a subset of TWIST samples to reduce the view sharing, the existing compressed sensing algorithm cannot be used because of severe coherent aliasing artifacts. The purpose of this research is to enhance the temporal resolution of TWIST by reducing the view sharing. In particular, we show that the recently proposed ALOHA algorithm^{3, 5} can provide significant improve temporal resolution without sacrificing the spatial resolution.

Theory

Suppose that a signal, $y(\mathbf{r})$, can be presented as sum of Dirac. We will relax this assumption later for transform domain sparse signals. Then, we can find an annihilating function of the signal, $h(\mathbf{r})$, such that

$$h(\mathbf{r}) \left(\sum_i a_i \delta(\mathbf{r} - \mathbf{b}_i) \right) = 0.$$

This property can be represented in a discrete convolution operation in k-space, which results in the matrix-vector multiplication:

$$\mathbf{H} \{\hat{\mathbf{y}}\} \hat{\mathbf{h}} = 0,$$

where $\mathbf{H} \{\hat{\mathbf{y}}\}$ is the 2-D Hankel matrix constructed from $\hat{\mathbf{y}}(\mathbf{k})$ and $\hat{\mathbf{h}}$ is the vectorized 2-D annihilating filter in k-space. If the underlying spatial domain signal is sparse, we can show that the Hankel matrix organized from k-spaces is rank-deficient³. Moreover, the annihilation property can be found from the inter-coil relationship in parallel MRI. However, the signals in the image domain are not generally sparse, but can be sparsified using transforms such as wavelets whose spectrum is $\hat{\phi}(\mathbf{k})$. In this case, the corresponding Hankel matrix can be constructed after multiplying $\hat{\phi}(\mathbf{k})$ to the k-space data³. In this study, we applied Haar wavelet weighting to $k_y - k_z$ data to construct Hankel matrix. Thanks to the wavelet weighting, the associated low-rank Hankel matrix completion can be performed using pyramidal decomposition as shown in Fig. 1, which reduces the computational complexity and improves the noise robustness³.

Method

Two sets of 3D DCE data, one for brain and the other for carotid and cerebral vessel imaging, were obtained using Siemens 3T Verio scanners. The TWIST sampling pattern for brain is shown in Fig. 2(a). The imaging parameters are as following: repetition time (TR) 2.81 ms, echo time (TE) 1.04 ms, 192x252x40 matrix size, 32 coils and 60 time frames. 1-D GRAPPA with 25 autocalibration lines was used. The TWIST sampling pattern for carotid and cerebral vessels is shown in Fig. 2(c). The imaging parameters are as follows: TR 2.5 ms, TE 0.94 ms, 159x640x80 matrix size, 16 coils and 30 time frames. 2D GRAPPA with 24x24 ACS regions was used. In addition, the partial Fourier was applied, so only 63% of data was acquired. The view sharing for GRAPPA for brain and carotid imaging are illustrated in Fig. 2(a)(c), respectively; whereas the corresponding view sharing scheme for ALOHA are shown in Fig. 2(b)(d).

The reduced view sharing in ALOHA causes irregular sampling pattern, so GRAPPA cannot be used for reconstruction. Furthermore, the conventional compressed sensing (CS)⁴ cannot be applied due to severe aliasing artifacts. Therefore, we employed the ALOHA, where the cost function is given by

$$\begin{aligned} \min_{\mathbf{Y}} \quad & \|\mathbf{Y}\| \\ \text{subject to } \quad & \mathbf{Y} = [\mathbf{H} \{\hat{\mathbf{M}}_1\} \cdots \mathbf{H} \{\hat{\mathbf{M}}_C\}], \\ & \hat{\mathbf{m}}_i(\mathbf{k}) = \hat{\phi}(\mathbf{k}) \hat{\mathbf{y}}_i(\mathbf{k}), \quad \mathbf{k} \in \Omega, \end{aligned}$$

where $\hat{\mathbf{M}}_i$ is a matrix constituted from samples of $\hat{\mathbf{m}}_i(\mathbf{k})$, and $\hat{\phi}(\mathbf{k})$ represents the wavelet spectrum that is used for weighting. $\hat{\mathbf{y}}_i(\mathbf{k})$ denotes the k-space measurement from the i -th coil, and Ω is the k-space indices. We used ADMM algorithm for minimization of this interpolation method^{3, 5}.

Result

ALOHA reconstruction images of both data set showed better temporal resolution than the conventional reconstruction images as shown Fig. 3 and Fig. 4. The propagation of contrast agent was gradually occurred in ALOHA reconstruction. Moreover, the propagation of the contrast agent in ALOHA reconstruction closely follows the best temporal resolution of the low resolution reconstruction without view sharing, which was not the case in GRAPPA. The improvement of temporal resolution was also evidenced by the comparison of the full width at half maximum (FWHM) in Fig. 4(b).

Discussion and Conclusion

In this paper, we proposed a novel TWIST reconstruction using recently proposed low rank k-space interpolation method called ALOHA. The in vivo results showed the significant improvement of temporal resolution. ALOHA turned out to be good for reduced view sharing, because the transform domain sparsity and the annihilation property from the inter-coil relationship are fully utilized simultaneously.

Acknowledgements

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References

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Figures

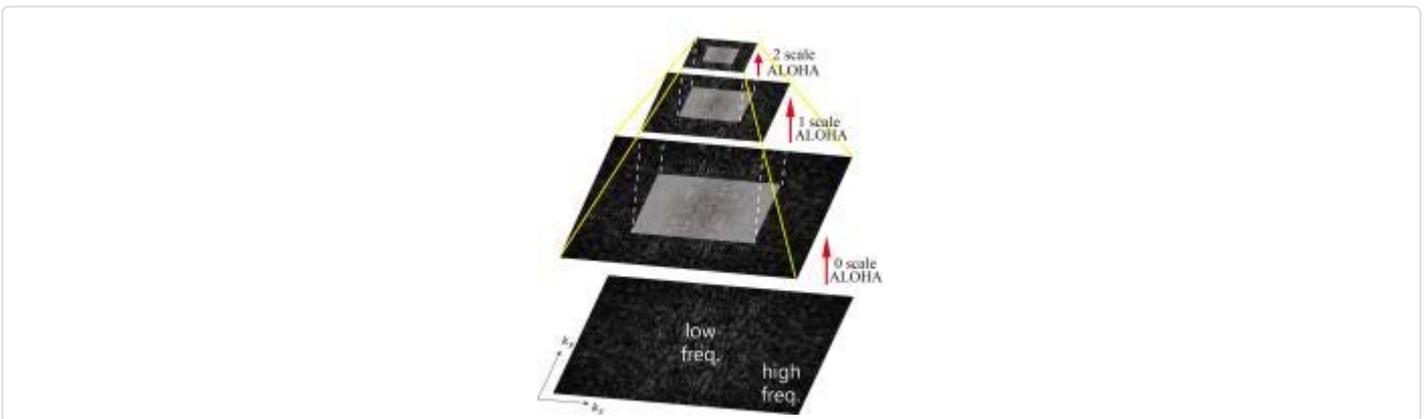


Fig 1. Three steps of the pyramidal decomposition in ALOHA reconstruction.

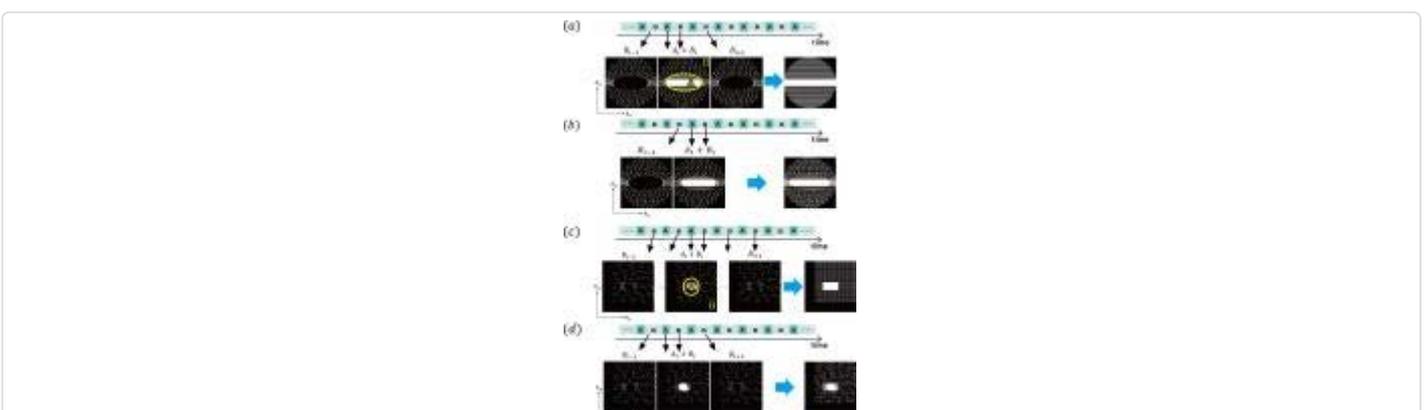


Fig 2. View sharing scheme. The center and periphery of k-space data are indicated by A and B, respectively. (a) Standard scheme for 1D GRAPPA reconstruction. (b) Proposed scheme of brain data for ALOHA. (c) Standard scheme for 2D GRAPPA reconstruction. (d) Proposed scheme of carotid and vessel data for ALOHA.

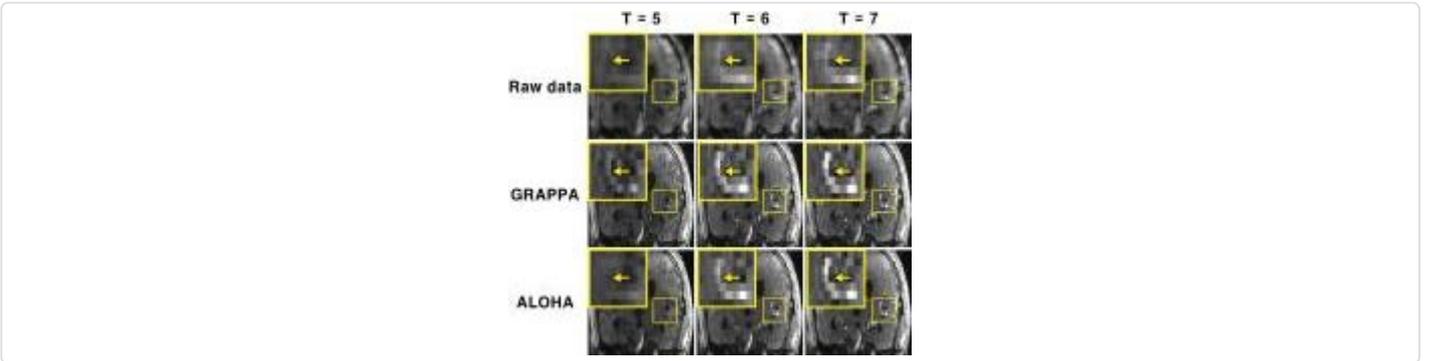


Fig 3. (Upper) Low resolution image of brain data without view sharing. (Middle) GRAPPA reconstruction. (Bottom) ALOHA reconstruction. Enlarged yellow boxes indicate flow of contrast agent.

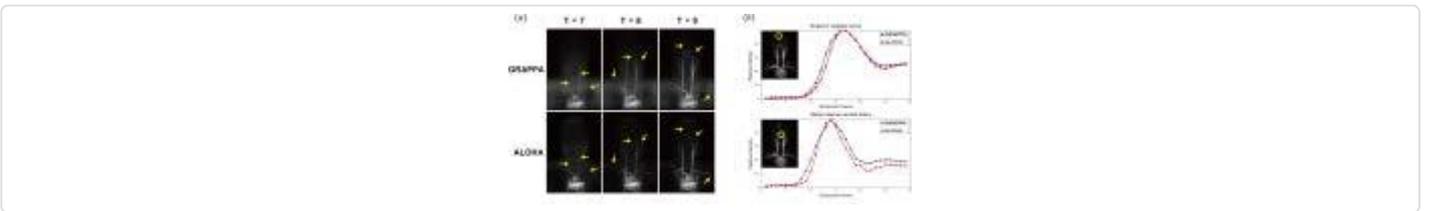


Fig 4. (a) Subtracted MIP images of carotid and cerebral vessel using standard (upper) and ALOHA (bottom) reconstruction. Yellow arrows indicate the differences. (b) Plots of relative temporal intensity of superior sagittal sinus and distal internal carotid artery in reconstruction.