

# Compressed Sensing Pulse-Echo Mode THz Tomography

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**Abstract**—This paper proposes a pulse-echo mode terahertz tomography technique for endoscopy applications. It can be shown that the sensing matrix of the pulse-echo mode acquisition has sufficiently small mutual coherence that allows the use of recently proposed compressive sensing theory in signal processing area. More specifically, if the target is sparse, then reconstruction algorithm such as the basis pursuit or greedy method can be applied to reduce the number of measurements and accelerate the acquisition process. Simulation results demonstrate that quite accurate reconstruction can be obtained from very limited number of measurements where the conventional Kirchhoff migration methods fails.

## I. INTRODUCTION AND BACKGROUND

TERAHERTZ imaging (T-ray imaging) has many advantages in bio-imaging applications since the molecular vibration mode corresponds to THz regime. Several fast imaging methods have been proposed, such as T-ray reflectance imaging[1], interferometric approaches[2], diffraction tomography[3], and computed tomography[3].

In this paper, we propose a novel pulse-echo mode tomography whose acquisition time can be significantly accelerated using compressed sensing [4]. In pulse echo mode, the relationship between the reflected wave function and target's refractive index function can be described by :

$$\psi_s(r; k) = k^2 S(k) \int dr' f(r) g_k^2(r|r') \quad (1)$$

where  $\psi_s$  is a scattered field,  $r$  denotes the source/detector pair position,  $k = \omega/c$  is a wavenumber and  $g(r|r')$  is Green's function in free space. In Eq.(1), we assume that the refractive index is not a function of wavenumber, which may be relaxed later. Eq.(1) can be discretized into a matrix equation :

$$y^{(n)} = G^{(n)} f, \quad n=1 \dots N \quad (2)$$

where  $y^{(n)}$  is  $\psi_s / k^2 S(k)$ ,  $S(k)$  is the spectrum of a source,  $G$  is  $g_k^2(r|r')$  and (n) means the index of measurements. The mutual coherence( $\mu$ ) for (2) can be shown

$$\mu \stackrel{\text{def}}{=} \max_{j \neq k} \left| \langle G_j, G_k \rangle \right| = \#\{\text{collinear measurements}\} / N \quad (3)$$

where  $G_j$  is the column of  $G$  matrix,  $\langle \rangle$  is the inner product operator, and  $||$  is the L0 norm. Hence, mutual coherence( $\mu$ ) is significantly small if the detector positions are not collinear.

Therefore, Eq.(3) can be solved by algorithm for the sparse approximation from compressive sensing. According to compressed sensing theory, the small coherence allows maximal acceleration by reducing the number of measurements. As a reconstruction algorithm, we employ the simultaneous

orthogonal matching pursuit (SOMP) since it is very easy to use.

## II. RESULTS

In our simulation, the imaging field of view(FOV) is defined as  $1.7\text{cm} \times 1.7\text{cm} \times 1.7\text{cm}$  and the detector/source pair is moving across the xy-plane, i.e.  $z=0$ . Total number of voxel is  $17^3$  and one voxel size is  $1\text{mm} \times 1\text{mm} \times 1\text{mm}$ .

We set the bandwidth as 2THz. We use the source( $S(k)$  in Eq.(2)) as a impulse which has uniform values over all frequency domain. Also we add the 20dB noise in time domain.

Multiple point targets are to form the letter 'THz' in 3D space. This phantom consists of 72 non-zero values. The number of measurement is 40 whose positions are overlaid at the bottom of Fig.1(a). When we reconstructed using S-OMP, it shows a good result (Fig.1). However, the conventional Kirchhoff migration methods[5] fails as shown in Fig.1(b).

In conclusion, we propose a novel pulse-echo mode THz imaging methods which can be significantly accelerated using compressive sensing. Simulation results confirm our theory.

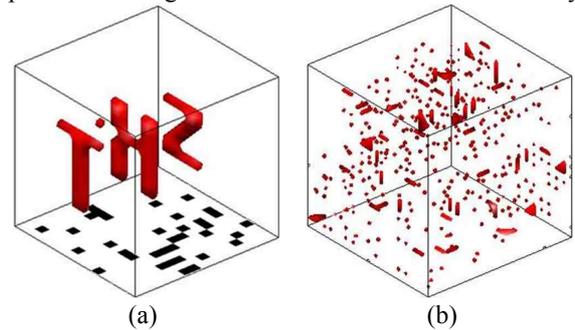


Fig.1. (a) Reconstruction using S-OMP algorithm using 40 waveforms. (b) Reconstruction using Kirchhoff migration using 40 waveforms. The source/detector positions are the same.

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