MMSE OPTIMAL NON-LOCAL MOTION COMPENSATED K-T FOCUSS FOR COMPRESSED SENSING CARDIAC CINE IMAGING

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ABSTRACT

A non-local motion compensation algorithm combined with k-t FOCUSS for high resolution compressed sensing cardiac cine imaging is proposed in this paper. While conventional non-local means algorithms use self-similarity, our method collects similar blocks from another reference frame that is obtained during diastole phase. We show that this non-local motion compensation is optimal in MMSE sense, if the probability for each candidate patch is uniform. Experimental results show that the proposed algorithm clearly reconstructs important edge boundaries in cardiac cine imaging and provides significant performance improvement compared to the existing motion compensated k-t FOCUSS.

Index Terms— cardiac cine imaging, compressed sensing, non-local motion compensation, non-local means, k-t FOCUSS, ME/MC

1. INTRODUCTION

In MRI, data collected at a scanner is a spatial Fourier transform of an object image. Hence, in order to obtain an image without aliasing, k-space samples should satisfy the Nyquist sampling criterion. However, MR imaging is an inherently slow imaging modality since it is designed to acquire 2-D (or 3-D) k-space data through 1-D free induction decay or echo signals. Hence, reconstruction for reduced k-space measurement is required.

Compressed sensing (CS) tells us that the perfect reconstruction is possible as long as the nonzero support in transform domain is sparse and sampling basis are incoherent [1]. By exploiting that dynamic MRI can be sparsified due to the temporal redundancy, we have demonstrated successful application of CS for cardiac imaging [2]. In particular, more accurate prediction using motion estimation/compensation [2] or data-driven optimal temporal sparsifying transforms [3] have proven to be quite effective in many dynamic compressed sensing algorithms. However, despite their successes to some extent, there still remain considerable artifacts in edge area when the acceleration factor increases.

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Recently, dictionary learning algorithms have been investigated quite extensively among CS community. For example, in K-SVD [4], patch based training data are used to construct an overcomplete dictionary from which sparse combination represents a signal. In a slight variation, non-local means algorithm [5] searches dictionary patches and calculates the linear weighting to estimate current pixel values. In BM3D [6], similar 2-D patches of the image are grouped into 3-D arrays, and so-called collaborative filtering using hard-thresholding and Wiener filtering, is performed for noise removal. In MR application, Ravishankar and Bresler applied K-SVD algorithm for static MRI reconstruction [7], whereas Akçakaya et al. applied BM3D collaborative filtering for cardiac MR application [8].

The procedures of non-local filtering, BM3D, and K-SVD algorithm share the same commonality in that these methods exploit self-similarity within images. However, these idea can be also extended to motion estimation and compensation scheme, where similarity patch comes from a reference frame rather than within the same frame. In particular, in cardiac cine imaging, reference frame can be obtained from diastole phase where the cardiac volume is relatively stationary. Hence, using this frame as a reference to generate dictionary patches, we propose a non-local motion compensation algorithm. Another important observation is that the non-local motion compensation algorithm can be interpreted as a minimum mean square error (MMSE) estimator for the best patch. This interpretation explains the significant performance improvement of the proposed method over the existing motion compensated k-t FOCUSS algorithm [2].

2. NON-LOCAL MOTION COMPENSATION

In this section, a novel block matching algorithm named as "non-local motion compensation" is described. Let us assume a measurement model:

$$\mathbf{y} = \mathbf{x} + \boldsymbol{\epsilon}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^N$ is a measurement, $\mathbf{x} \in \mathbb{C}^N$ is an original signal and $\boldsymbol{\epsilon} \in \mathbb{C}^N$ is a measurement error. In our non-local motion compensation, a subvector $\mathbf{x}_{I_i}$ at the index set I_i is

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estimated by minimizing the following cost function

$$\mathbf{c}(\mathbf{x}) = \sum_{j\in\mathcal{N}_i} \theta(i,j)(\mathbf{y}^r_{I_j} - \mathbf{x})^H(\mathbf{y}^r_{I_j} - \mathbf{x}), \tag{2}$$

where $\mathcal{N}_i$ denotes the collection of index sets in a reference frame $\mathbf{y}^r$ where the similar patches can be found, and $\mathbf{y}^r_{I_i}$ denotes the patch from the reference frame at the index set I_i. Eq. (2) can be regarded as an extension of a point estimation framework in the conventional non-local means estimation if the reference image is same as the current image. The solution for the optimization problem (2) is given by minimizing the cost function with respect to $\mathbf{x}$. More specifically, we have

$$\hat{\mathbf{x}}_{I_i} = \frac{\sum_{j\in\mathcal{N}_i}\theta(i,j)\mathbf{y}^r_{I_j}}{\sum_{j\in\mathcal{N}_i}\theta(i,j)} = \sum_{j\in\mathcal{N}_i} p(i,j)\mathbf{y}^r_{I_j}, \tag{3}$$

where

$$p(i,j) = \frac{\theta(i,j)}{\sum_{l\in\mathcal{N}_i}\theta(i,l)}. \tag{4}$$

In practice, $\mathcal{N}_i$ is restricted within a fixed set of windows assuming that similarity blocks are mainly localized in space. Moreover, rather than using $\mathcal{N}_i$ as a fixed size, we define the index set such that $\mathcal{N}_i = \{j : ||\mathbf{y}_{I_i} - \mathbf{y}^r_{I_j}|| < \epsilon\}$ for a given threshold values within a window. Now, the remaining issue in non-local motion compensation is to find the weighting parameter $\theta(i,j)$. We use the same formulation as in non-local means algorithm [5] such that

$$\theta(i,j) = e^{-\frac{||\mathbf{y}_{I_i} - \mathbf{y}^r_{I_j}||^2}{2B\lambda^2}} \tag{5}$$

where $B = |I_i|$ is the size of a patch and λ is the smoothing parameter.

Now, our goal is to show that the non-local motion compensation corresponds to minimum mean square error (MMSE) estimator if the reference patches are assumed to have identical probability mass prior distribution. More specifically, let the prior distribution be given by

$$p(\mathbf{x}) = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\delta(\mathbf{x} - \mathbf{y}^r_{I_j}). \tag{6}$$

If our measurement $\mathbf{y}_{I_i}$ follows the Gaussian distribution

$$p(\mathbf{y} = \mathbf{y}_{I_i}|\mathbf{x}) \propto e^{-\frac{||\mathbf{y}_{I_i} - \mathbf{x}||^2}{2B\lambda^2}}, \tag{7}$$

then the posterior distribution is given by

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{\int p(\mathbf{y}|\mathbf{x})p(\mathbf{x})d\mathbf{x}}, \tag{8}$$

and the MMSE estimator of $\mathbf{x}_{I_i}$ for $\mathbf{y} = \mathbf{y}_{I_i}$ is

$$\begin{aligned}\hat{x}_{I_i,\text{MMSE}} &= \int \mathbf{x}p(\mathbf{x}|\mathbf{y} = \mathbf{y}_{I_i})d\mathbf{x} = \frac{\int \mathbf{x}p(\mathbf{y}|\mathbf{x})p(\mathbf{x})d\mathbf{x}}{\int p(\mathbf{y}|\mathbf{x})p(\mathbf{x})d\mathbf{x}} \\ &= \frac{\sum_{j\in\mathcal{N}_i}\mathbf{y}^r_{I_j}e^{-\frac{||\mathbf{y}_{I_i} - \mathbf{y}^r_{I_j}||^2}{2B\lambda^2}}}{\sum_{j\in\mathcal{N}_i}e^{-\frac{||\mathbf{y}_{I_i} - \mathbf{y}^r_{I_j}||^2}{2B\lambda^2}}} = \sum_{j\in\mathcal{N}_i} p(i,j)\mathbf{y}^r_{I_j}, \end{aligned} \tag{9}$$

which is equivalent to the non-local motion compensation in (3) and (4).

Since non-local means algorithm [5] and non-local motion compensation are identical in computing weights, both of them can be interpreted as MMSE optimal. However, there are two main differences between them. First, in non-local means, the center pixel of a processed block $\mathbf{x}$ is replaced with the center pixel of the estimated block $\hat{\mathbf{x}}_{I_i}$, while the entire block is replaced with the estimated blocks in the non-local motion compensation. Second, non-local means searches for similar blocks within image itself to impose "self-similarity". However in non-local motion compensation, similar blocks are retrieved in another reference image, which is usually in high resolution.

3. NON-LOCAL MOTION COMPENSATED K-T FOCUSS

In cardiac cine imaging, there are significant temporal redundancies along temporal direction. In particular, the spectral support in x-f space is very sparse due to the periodic motion where x denotes the spatial dimension along phase encoding direction and f denotes the temporal Fourier frequency. Furthermore, if we have good prediction $\bar{\mathbf{x}}$ of unknown x-f signal $\mathbf{x}$, then we can estimate the x-f image as $\mathbf{x} = \bar{\mathbf{x}} + \Delta\mathbf{x}$ where the residual $\Delta\mathbf{x}$ is obtained by solving the following compressed sensing problem :

$$\min||\Delta\mathbf{x}||_1, \quad \text{subject to } ||\Delta\boldsymbol{v} - \mathbf{F}\Delta\mathbf{x}||_2 \le \epsilon \tag{10}$$

where $\mathbf{F}$ denotes the 2-D Fourier transform that convert x-f signal $\Delta\mathbf{x}$ to k-t measurement residual $\Delta\boldsymbol{v}$, where $\Delta\boldsymbol{v} = \boldsymbol{v} - \mathbf{F}\bar{\mathbf{x}}$. The k-t FOCUSS solves the optimization problem (10) using the following reweighted norm approach [3] :

$$\min ||\Delta\mathbf{q}_l||_2, \quad \text{subject to } ||\Delta\boldsymbol{v} - \mathbf{F}\mathbf{W}_l\Delta\mathbf{q}_l||_2 \le \epsilon, \tag{11}$$

where $\Delta\mathbf{x}_l = \mathbf{W}_l\Delta\mathbf{q}_l$ and $\mathbf{W}_l$ is a diagonal weighting matrix at the l-th iteration. Here, $\mathbf{W}_l$ is updated as follows:

$$\mathbf{W}_l = \text{diag}[|\Delta\mathbf{x}_l(1)|^p \cdots |\Delta\mathbf{x}_l(M)|^p], \quad 1/2 \le p \le 1. \tag{12}$$

Setting $p = 0.5$, k-t FOCUSS obtains an l_1 minimization solution. Using a Lagrangian multiplier, the solution of (11):

$$\mathbf{x}_{l+1} = \bar{\mathbf{x}}_{l+1} + \boldsymbol{\Theta}_l\mathbf{F}^H(\mathbf{F}\boldsymbol{\Theta}\mathbf{F}^H + \lambda\mathbf{I})^{-1}(\boldsymbol{v} - \mathbf{F}\bar{\mathbf{x}}_{l+1}), \tag{13}$$

where $\boldsymbol{\Theta}_l = \mathbf{W}_l\mathbf{W}_l{}^H$ and $\bar{\mathbf{x}}_{l+1}$ is a prediction term.

In our previous work, motion compensated image frame is used to improve the performance of k-t FOCUSS encoding [2], i.e. :

$$\begin{aligned}&\bar{\mathbf{x}}_{l+1} \leftarrow \text{Motion compensated images from } \mathbf{y}^r \\ &\mathbf{x}_{l+1} = \bar{\mathbf{x}}_{l+1} + \underbrace{\boldsymbol{\Theta}_l\mathbf{F}^H(\mathbf{F}\boldsymbol{\Theta}_l\mathbf{F}^H + \lambda\mathbf{I})^{-1}(\boldsymbol{v} - \mathbf{F}\bar{\mathbf{x}}_{l+1})}_{\text{residual encoding}}.\end{aligned} \tag{14}$$

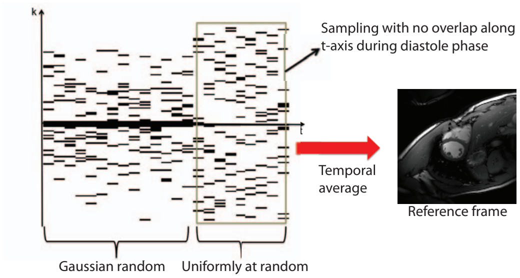

Fig. 1. Proposed sampling pattern.

More accurate prediction term into k-t FOCUSS promises better quality of reconstruction image because the accurate prediction leads to sparser residual in measurement space [2]. In ME/MC, just one matching block in reference frame which is closest to the searching block in the currently processed image is selected. Since non-local motion compensation outperforms the conventional ME/MC, we will incorporate the non-local motion compensation with k-t FOCUSS as a more accurate prediction algorithm.

4. EXPERIMENTAL RESULT

4.1. Methods

A cardiac cine data was acquired using a Siemens 3T whole-body MRI scanner. The acquisition sequence was bSSFP and prospective cardiac gating was used. FOV is $300 \times 300\, mm^2$, matrix size is 128×128 and the number of cardiac phase is 23.

The sampling pattern is shown in Fig. 1, which corresponds to 8x acceleration. We generated high quality reference frame from the data itself using the diastolic phase. More specifically, during the diastolic phase, the sampling patterns are chosen uniformly at random. Sampling points are judiciously designed not to be overlapped at the same position of the other frames during the diastole phase. Then we can obtain high resolution reference image just by directly adding diastole frames and taking the inverse Fourier transform. Outside the diastolic phase follow Gaussian random sampling patterns.

k-t FOCUSS algorithm was used to generate the intermediate quality reconstruction image. Then the patches in reference frame are used to perform the non-local motion compensation at each frame. The parameters used for similar block selection to meet high performance as well as admittable computational complexity was 4 by 4 block size, 17 by 17 search range, smoothing parameter $\lambda = 3.1$ and 1.5 times the normalized l_2 distance of the closest block as threshold to select index set $\mathcal{N}_i$. The increment of the processed block was 1. The scheme of non-local motion compensation is shown in Fig. 2. Here, estimated blocks were then overlapped because increment is smaller than block dimension. The overlapped

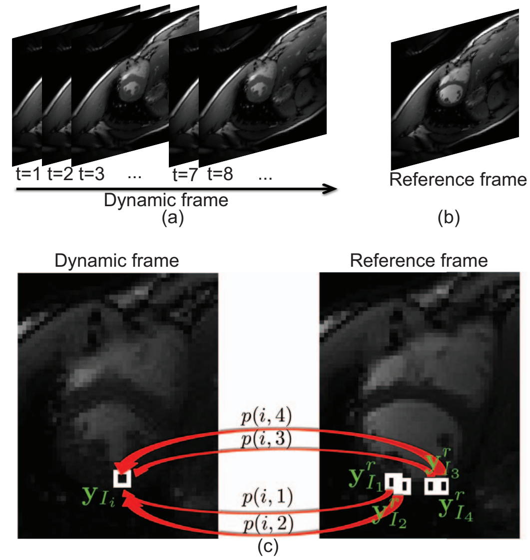

Fig. 2. Schematics of the nonlocal motion compensation. (a) Dynamic frames along time axis, and (b) reference frame from diastole phase. (c) A block in the currently processed frame is estimated by in an MMSE sense as a weighted average of similar blocks in reference frame, i.e. $\hat{\mathbf{x}}_{I_i} = \sum_{j \in \mathcal{N}_i} p(i,j)\mathbf{y}^r_{I_j}$.

blocks were averaged pixelwise. Then, the finally obtained motion compensated image is subtracted from the measurement data in the k-t measurement space to obtain residual. The final reconstruction image can be represented as a summation of the residual encoded image and non-local motion compensated image. Also, the whole procedure can be iterated. The procedure of the proposed algorithm is shown in Algorithm 1.

Algorithm 1 non-local motion compensated k-t FOCUSS for cardiac cine imaging

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Input : $\tilde{\mathbf{y}}$ - intermediate quality image reconstructed using k-t FOCUSS
$\mathbf{y}^r$ - reference image for high resolution diastole phase

Initialization : $\mathbf{x}_0 = \tilde{\mathbf{y}}$

Iteration : l = 0,1,2, ...

1. Find similar blocks for blocks of $\mathbf{x}_l$ from $\mathbf{y}^r$.
2. Update non-local motion compensation image $\bar{\mathbf{x}}_{l+1}$ using (3).
3. Perform k-t FOCUSS in (13) using $\bar{\mathbf{x}}_{l+1}$ for update of $\mathbf{x}_{l+1}$.

4.2. Results

Fig. 3 shows the reconstruction results at the down-sampling factor of 8. As can be seen in the results, both non-local

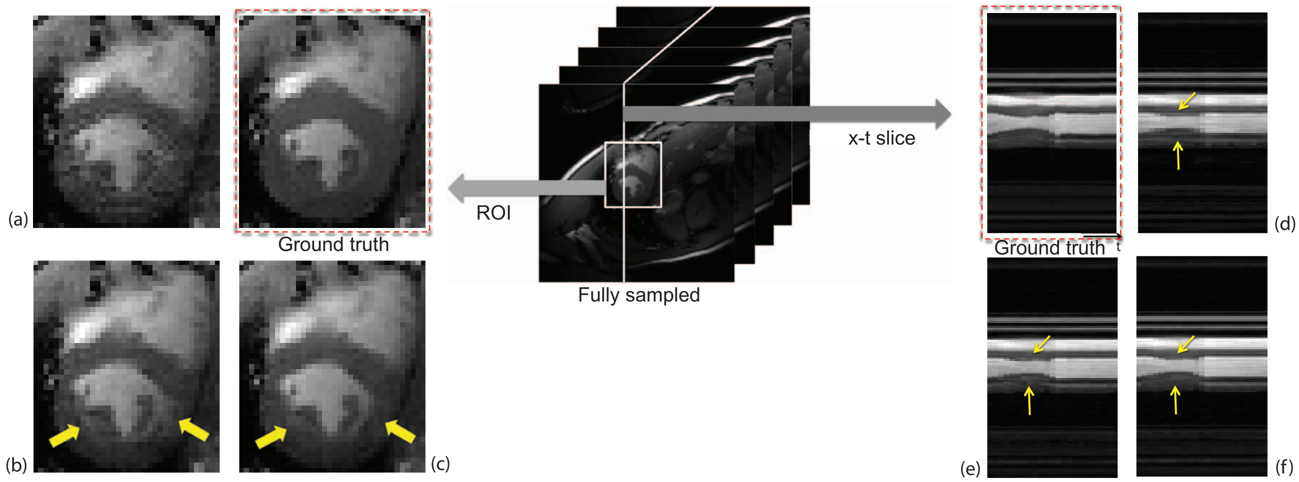

Fig. 3. Magnified view at heart region of reconstuction using (a) k-t FOCUSS, (b) k-t FOCUSS with ME/MC, and (c) k-t FOCUSS with nonlocal motion compensation. Recostruction x-t profile using (d) k-t FOCUSS, (e) k-t FOCUSS with ME/MC, and (f) k-t FOCUSS with nonlocal motion compensation. Nonlocal motion compensation shows clearer muscle and myocardium structure.

motion compensation and existing ME/MC improve the reconstruction quality of k-t FOCUSS. However, noticeable improvement can be observed along cardiac wall boundaries especially in non-local motion compensation. In nonlocal motion compensation result, left ventricular myocardium and muscles were clearly visible compared to that of the ME/MC. Similar results were obtained across all frames along heart wall (results not shown). Also in x-t slice view (Fig. 3), the nonlocal motion compensation shows the most clear reconstruction near the cardiac motion among them.

5. CONCLUSION

In this paper, improved motion compensated k-t FOCUSS algorithm using non-local motion compensation was proposed. In order to improve the quality of edge boundary of cardiac cine image, we proposed a non-local motion compensation algorithm which generates more accurate prediction images than the conventional block matching algorithm. Non-local motion compensation was shown MMSE optimal and experimental result showed that the proposed algorithm clearly reconstruct the important cardiac structures and improved over the conventional motion-compensated k-t FOCUSS.

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