KAIST Dept. of Bio and Brain Engineering

BiSPL Bio Imaging Signal Processing Lab

IEEE EMBS Summer School on Biomedical Imaging

Sparse and Deep Learning Approaches for Biomedical Image Reconstruction:

Part I: Compressed Sensing & Sparse Recovery

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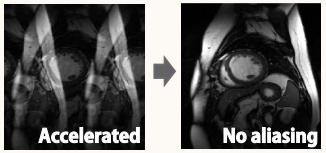
Bio-signal processing & Machine learning

Applied mathematics, signal processing & machine learning **for bio-medical imaging**

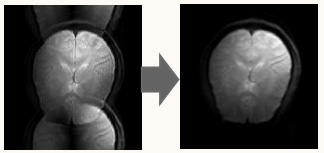
MRI Acceleration



Fast imaging using Compressed sensing

ISMRM09 recon Challenge winner! (k-t FOCUSS)

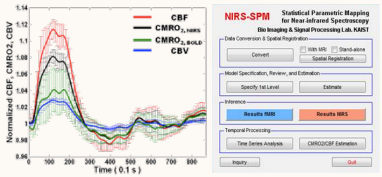
MRI Signal Processing

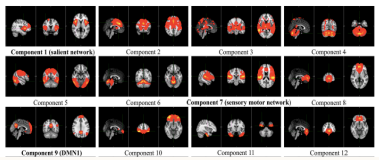


MR artifact removal using ALOHA

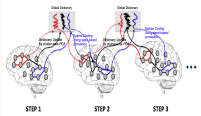
Neuro-Imaging (statistical tool development)

NIRS-SPM

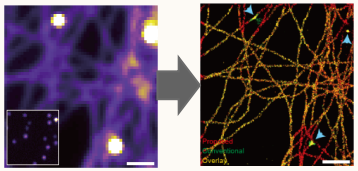




Sparse-SPM

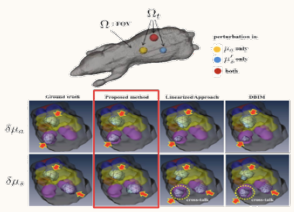


Optical Imaging

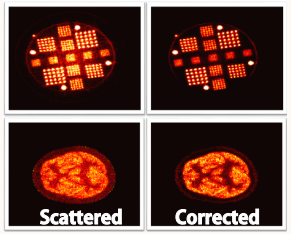


Super-resolution imaging (*FALCON*)

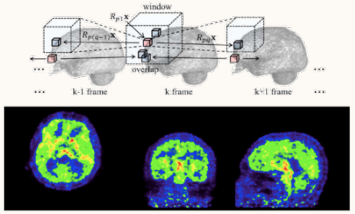
Diffuse optical tomography (DOT)



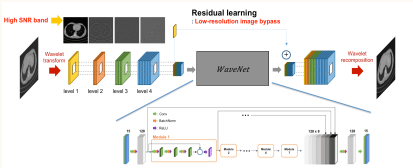
PET Reconstruction



Dynamic 4D PET reconstruction

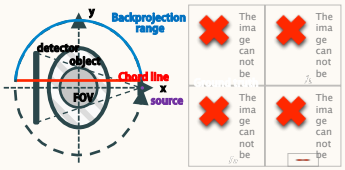


Machine Learning/Deep Learning for Bio-medicine

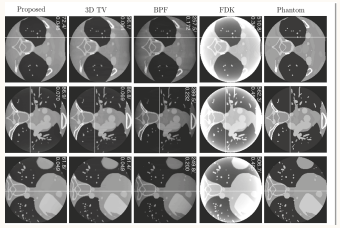


2016 AAPM Low-Dose CT Grand Challenge Winner (WaveNet)

x-ray CT Reconstruction



Interior Tomography

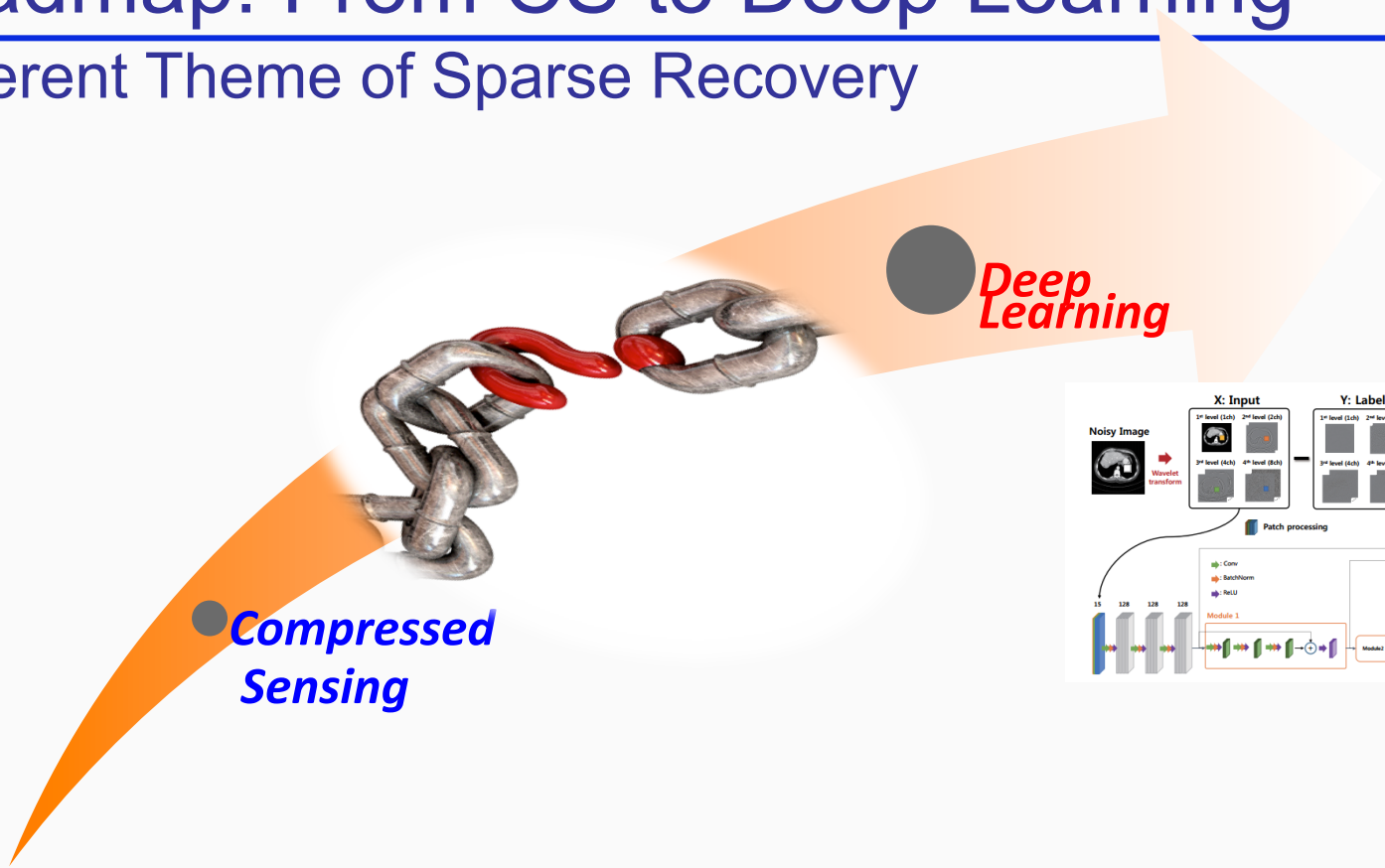


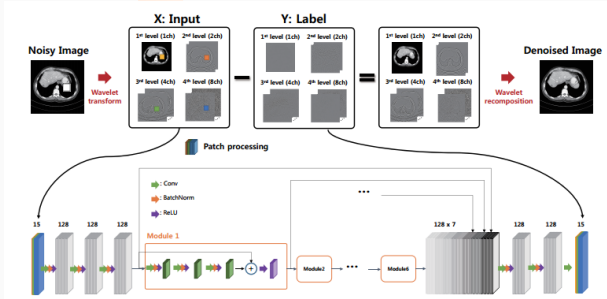
Course Overview

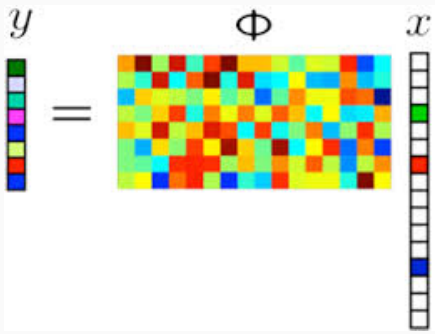
1. Introduction
2. Part I: Compressive Sensing
   - Motion-compensated CS
   - Learning-based CS
   - Multiple measurement vector (MMV) CS
3. Part II: Low-rank Hankel Matrix Approaches
4. Part III: Deep learning approaches
5. Open problems and outlook

Roadmap: From CS to Deep Learning

Coherent Theme of Sparse Recovery

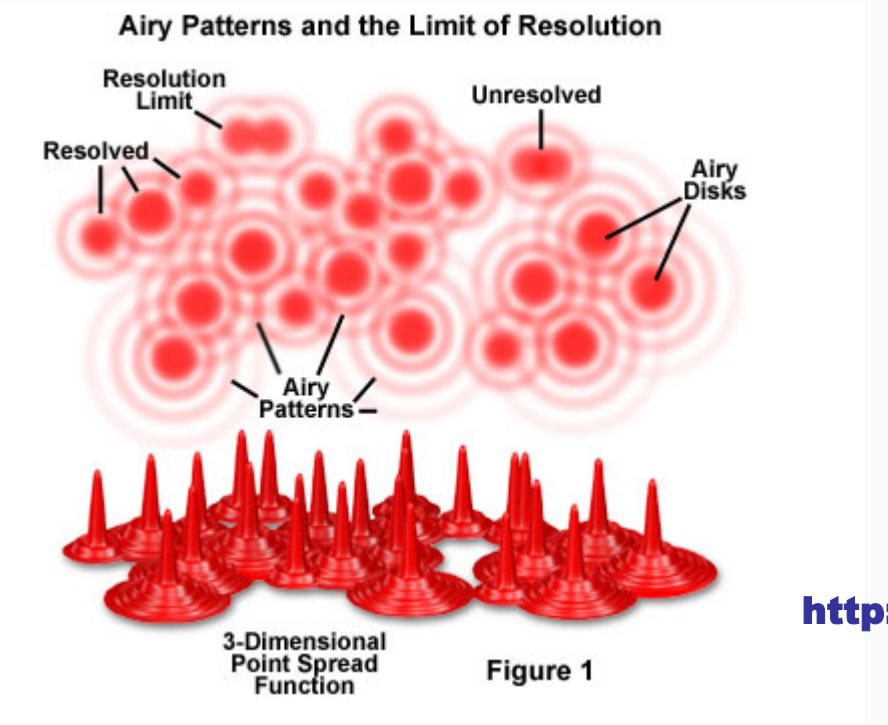


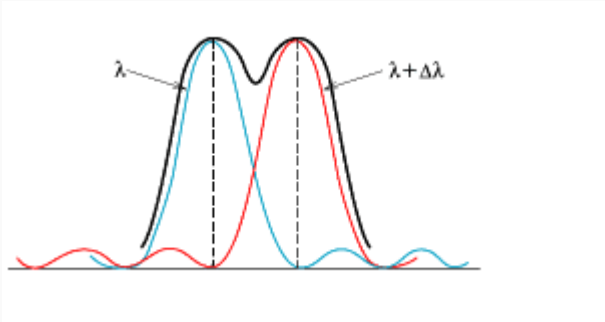




Resolution Limits in Medical Imaging

- Diffraction Limit





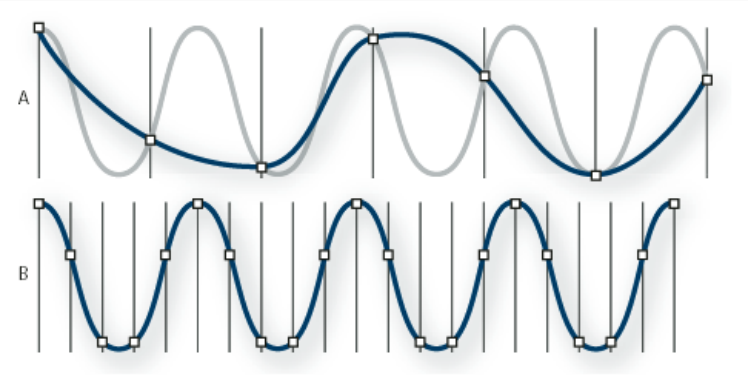
$$\text{Resolution} \simeq \frac{\lambda}{2NA}$$

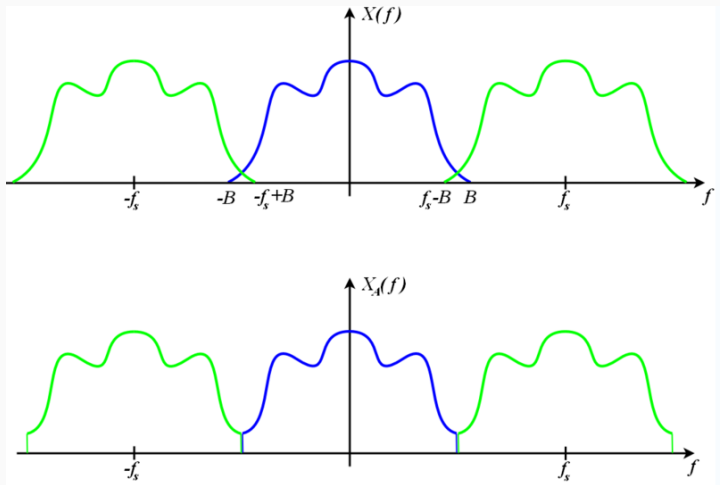
http://www.microscopyu.com/

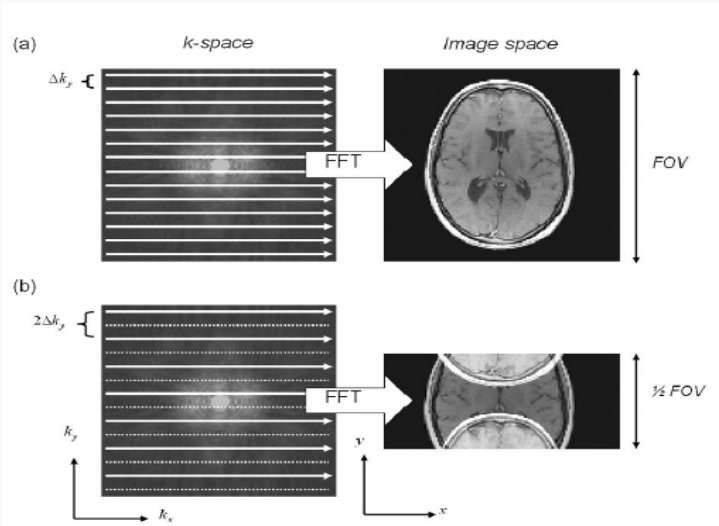
– Resolution limit for optics, x-ray, PET, and etc.

Resolution Limits in Medical Imaging

- Nyquist Sampling limit





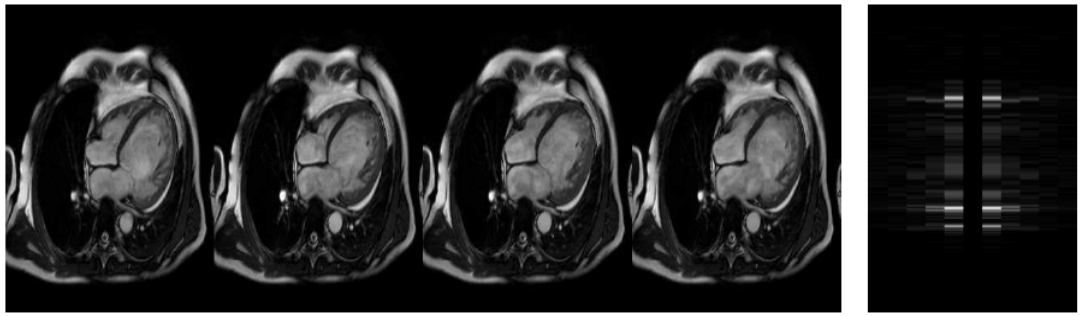


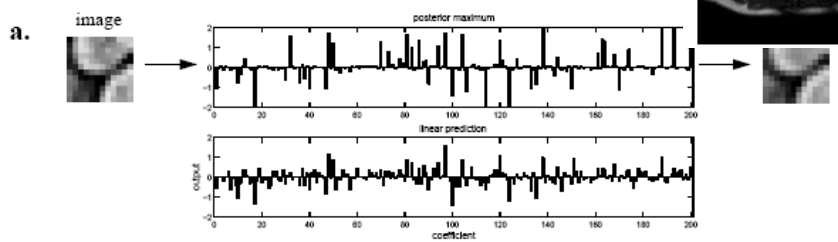
  - Nyquist requires the minimun two times sample rate w.r.t. signal bandlimit
  - Resolution limit for Fourier based imaging (MRI, etc.)

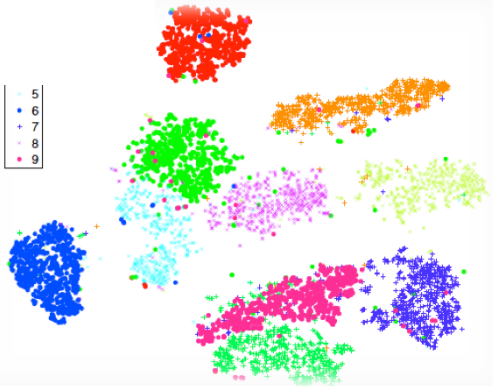
Law of Parsimony: Sparsity, Low-rank, LDM

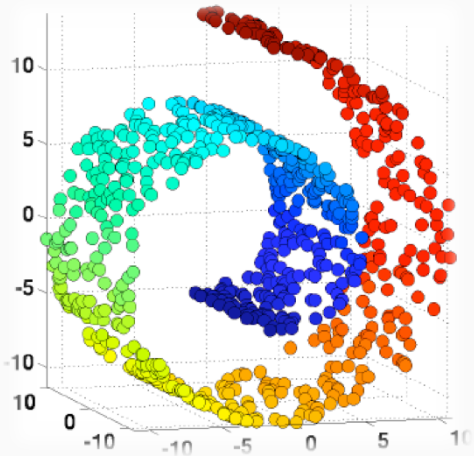
- Occam's Razor: law of parsimony by William Occam (14th century)

  Entities should not be multiplied beyond necessity.
- All things being equal, the simplest solution tends to be the best one.









Super-resolution Microscopy

The Nobel Prize in Chemistry 2014

Photo: Matt Staley/HHMI

Eric Betzig

Prize share: 1/3

© Bernd Schuller, Max-Planck-Institut

Stefan W. Hell

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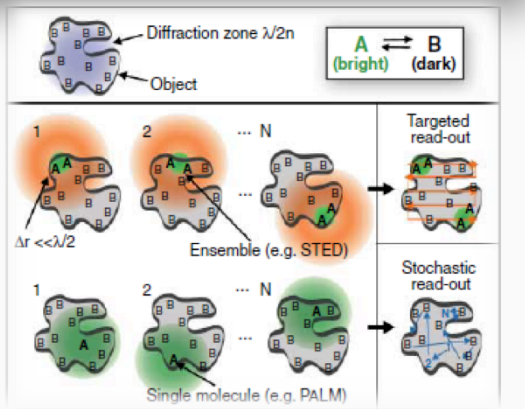
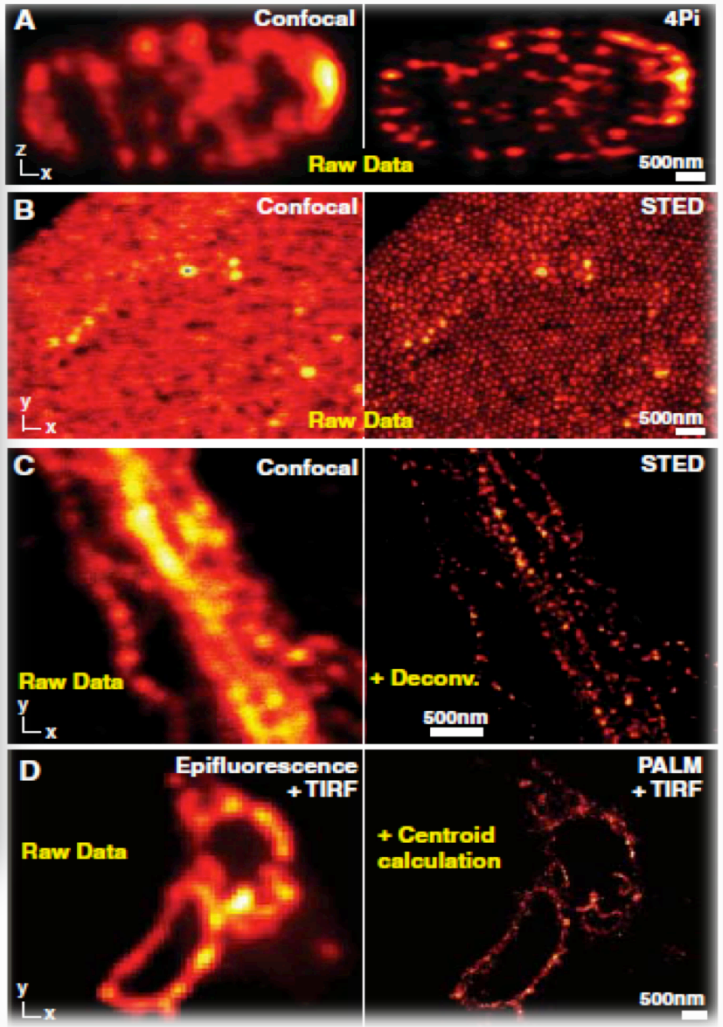
Photo: K. Lowder via Wikimedia Commons, CC-BY-SA-3.0

William E. Moerner

Prize share: 1/3

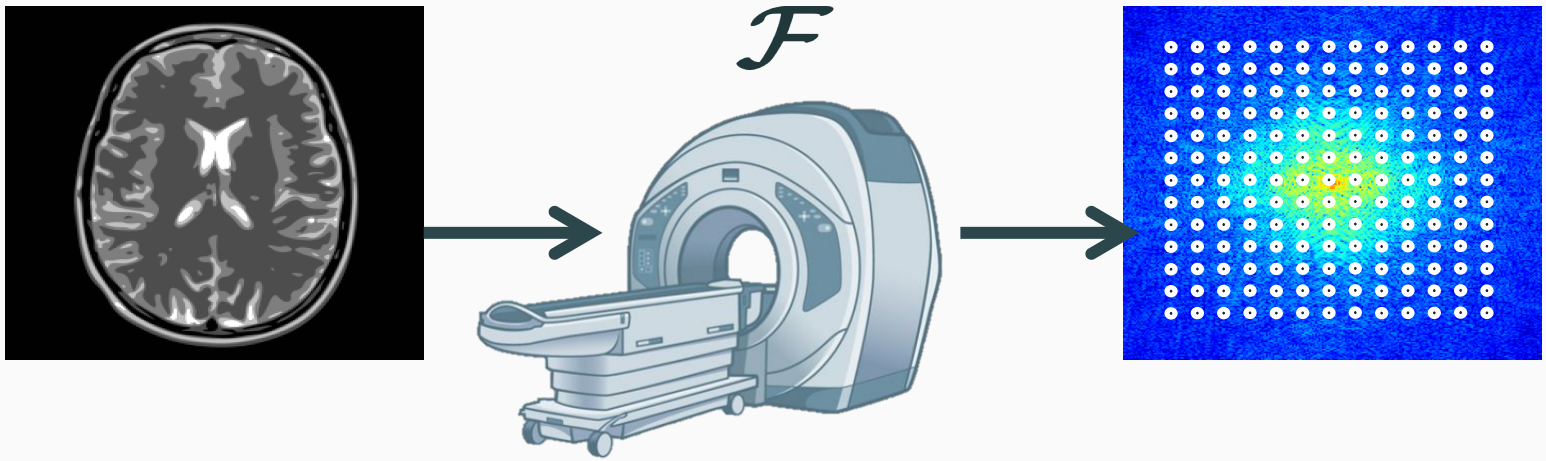




S. Hell et al, Science 2007.

Compressed Sensing MR Imaging

- Forward problem



- Sparse recovery problem

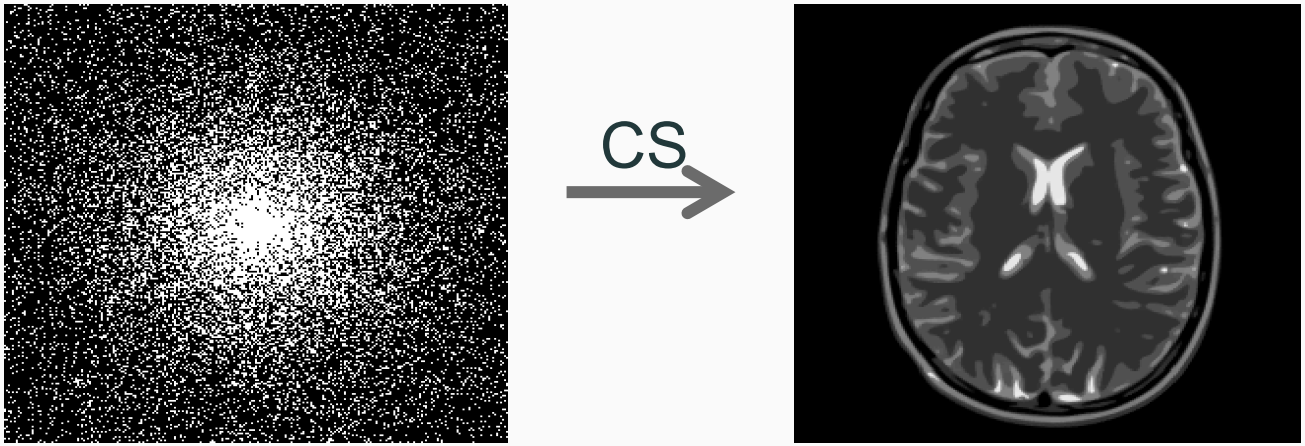
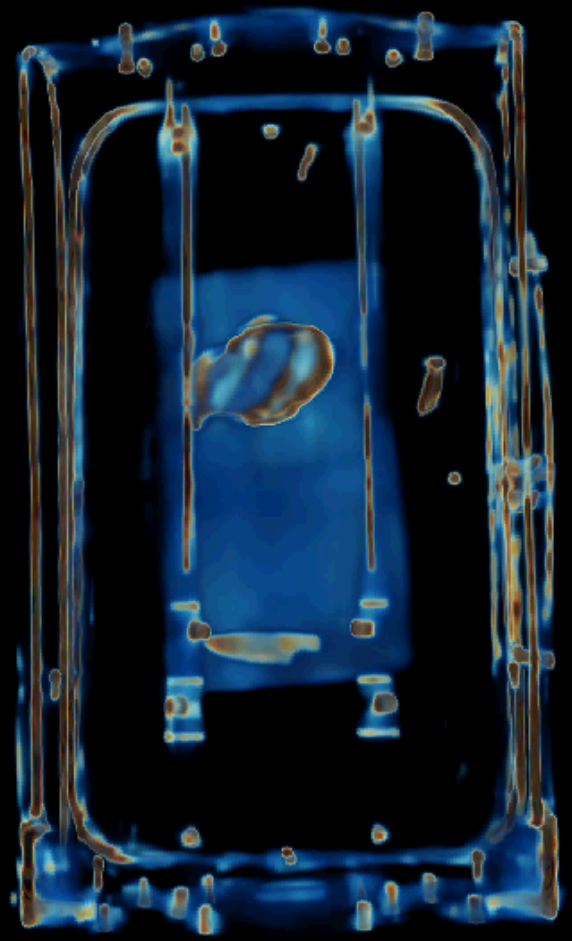


Figure courtesy of Mathews Jacob

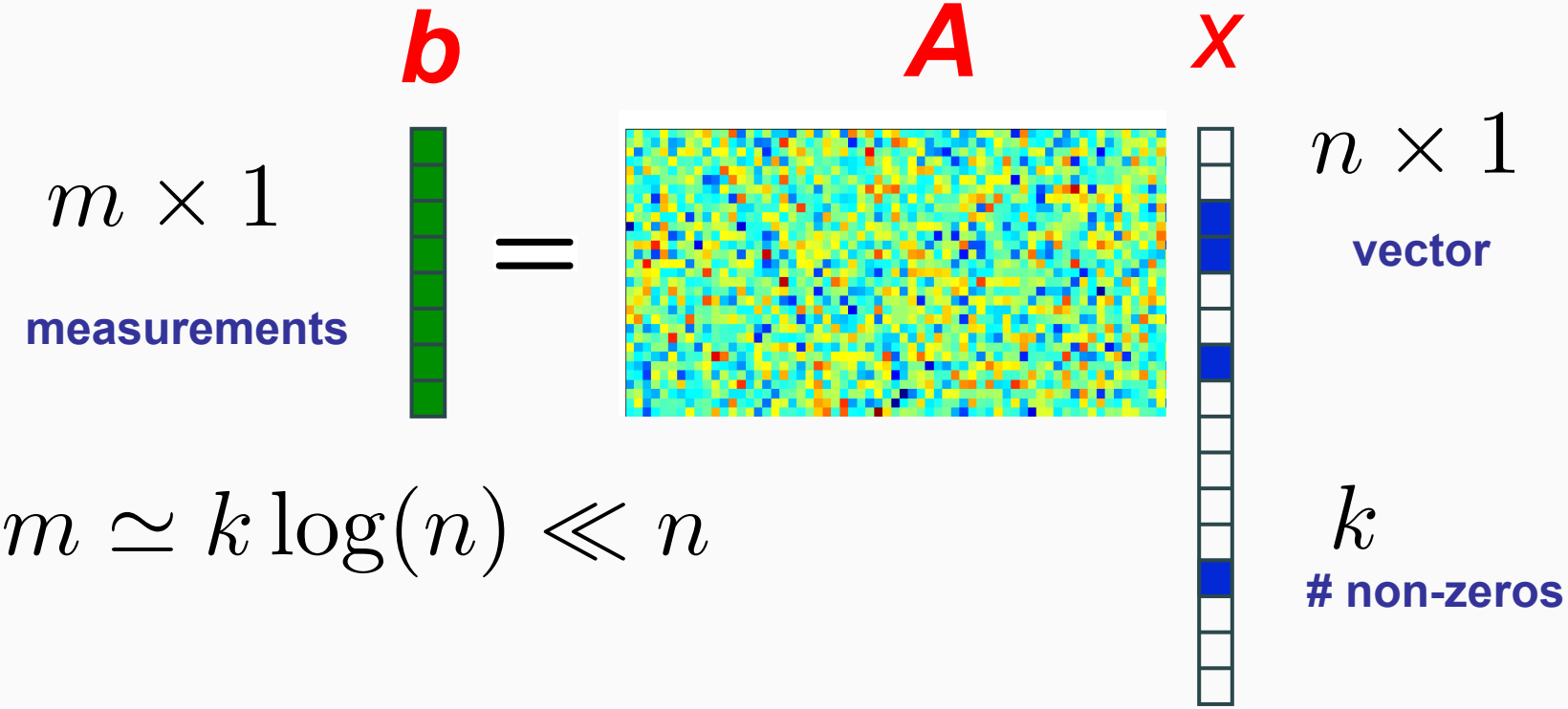
9 View CT Reconstruction using Deep Learning



COMPRESSED SENSING

Compressed Sensing (CS)

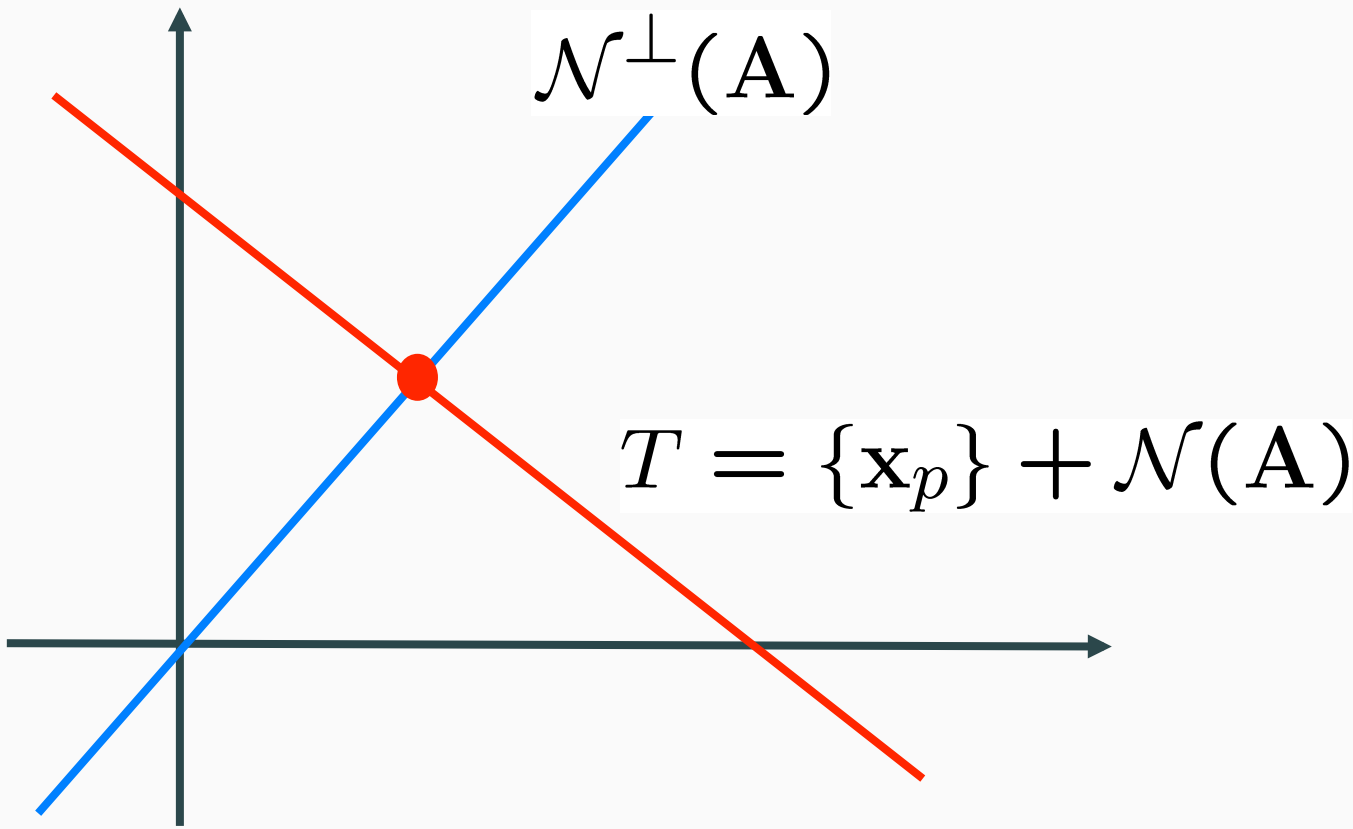
- Incoherent projection
- Underdetermined system
- Sparse unknown vector



$$m \simeq k \log(n) \ll n$$

Courtesy of Dr. Dror Baron

Min-norm Solution is Not sparse



L_0 Minimization

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|_0$$
$$\text{subject to} \quad \mathbf{b} = A\mathbf{x}$$

$$\|\mathbf{x}\|_0 = \{\#\text{ of non-zero elements of } \mathbf{x}\}$$

- Two fundamental questions
  - Uniqueness ?
  - Convex relaxation ?

Uniqueness and Spark

Definition

Given a matrix $A = [a_1, \cdots, a_n]$, the quantity $\mathrm{spark}(A)$ is the smallest possible number of linearly dependent columns of A.

Theorem (Donoho, Elad)

$\mathbf{x} \in \mathbb{R}^n$ *is the unique solution of the problem* $[P0]$ *if* $A\mathbf{x} = \mathbf{b}$ *and*

$$\|\mathbf{x}\|_0 < \frac{spark(A)}{2}.$$

L1 Convex Relaxation

$$\boxed{\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 \\ \text{subject to} \quad & \mathbf{b} = A\mathbf{x} \end{aligned}}$$

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

Intuition behind l1

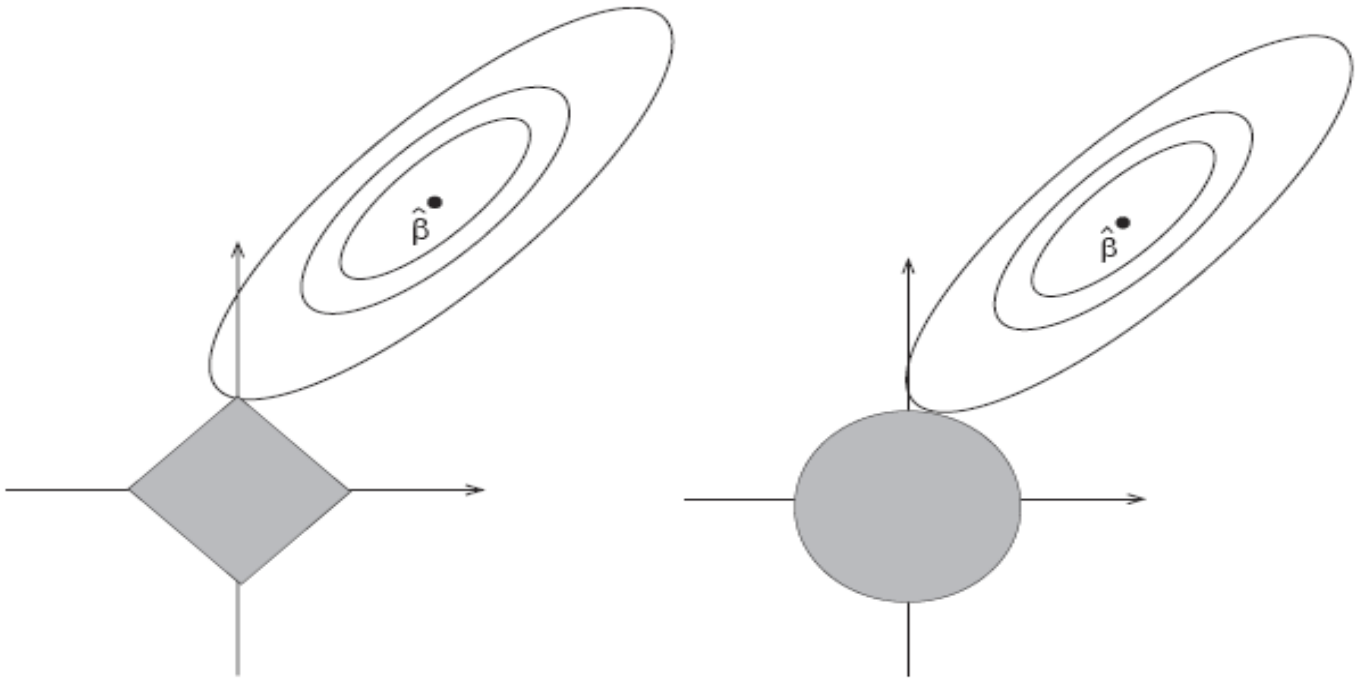


Figure 2: Estimation picture for the lasso (left) and ridge regression (right)

R. Tibshirani, "Regression Shrinkage and Selection via the Lasso", Jour. Royal Stat. Soc. B, 1996,

Sufficient Conditions for P1

Theorem (Donoho, Huo)

If $\mathbf{x} \in \mathbb{R}^n$ *is a k-sparse vector such that* $A\mathbf{x} = \mathbf{b}$ *and*

$$k < \frac{1}{2}\left(1 + \frac{1}{\mu(A)}\right)$$

or

$$k < \mu_{1/2}(A)$$

then $\mathbf{x}$ *is the unique solution to* $[P1]$ *and this solution is identical with the solution to* $[P0]$.

Theorem (Strohmer, Heath)

Let $A \in \mathbb{R}^{m \times n}$ *with normalized columns. The mutual coherence must satisfy*

$$\mu(A) \geq \sqrt{\frac{n-m}{m(n-1)}}.$$

Note : In the deterministic setting, we must obtain at least $O(k^2)$ samples.

RIP Conditions

- RIP has been developed for probabilistic argument .

Definition

A matrix $A \in \mathbb{R}^{m \times n}$ is said to have a k-restricted isometry property(RIP) if there is a constant $0 < \delta_k < 1$ such that

$$(1 - \delta_k)\|\mathbf{x}\|^2 \leq \|A\mathbf{x}\|^2 \leq (1 + \delta_k)\|\mathbf{x}\|^2 \quad [RIP]$$

for every k-sparse vector $\mathbf{x}$.

Theorem (Candes)

If $\mathbf{x} \in \mathbb{R}^n$ is a k-sparse vector such that $A\mathbf{x} = \mathbf{b}$ and A satisfies the RIP condition with

$$0 < \delta_{2k} < 1$$

then $\mathbf{x}$ is the unique solution to $[P0]$.

RIP as Sufficient Condition

Theorem (Candes)

Let $\mathbf{x} \in \mathbb{R}^n$ *is a* k*-sparse vector such that* $A\mathbf{x} = \mathbf{b}$. *If* A *satisfies the* [*RIP*] *with* $\delta_{2k} < \sqrt{2} - 1$ *then*

- *there is a unique* k*-sparse solution* $\mathbf{x}$ *consistent with* $\mathbf{b}$;
- *the problem* [$P1$] *has a unique solution;*
- *the solution to the* [$P1$] *is equal to* $\mathbf{x}$.

RIP: Robust Recovery

Suppose that the measurements are corrupted by bounded noise so that

$$\mathbf{b} = A\mathbf{x} + \mathbf{n}$$

where $\|\mathbf{n}\| \leq \epsilon$. In order to recover $\mathbf{x}$, we use the modified $L1$-problem

$$\min \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{b} - A\mathbf{x}\| \leq \epsilon. \qquad [P1']$$

Theorem (Candes)

Let $\mathbf{b}$ is the noisy measurements of $\mathbf{x} \in \mathbb{R}^n$ with noise $\|\mathbf{n}\| \leq \epsilon$. Let $\mathbf{x}^k$ denote the best k-sparse approximation of $\mathbf{x}$ and $\mathbf{x}'$ be the solution of $[P1']$. If A satisfies the RIP with $\delta_{2k} < \sqrt{2} - 1$, then

$$\|\mathbf{x} - \mathbf{x}'\| \leq \frac{C_1}{\sqrt{k}} \|\mathbf{x} - \mathbf{x}^k\|_1 + C_2 \epsilon$$

for some constants $C_1, C_2 > 0$.

Sparse Recovery Formulations

- Analysis Formulation

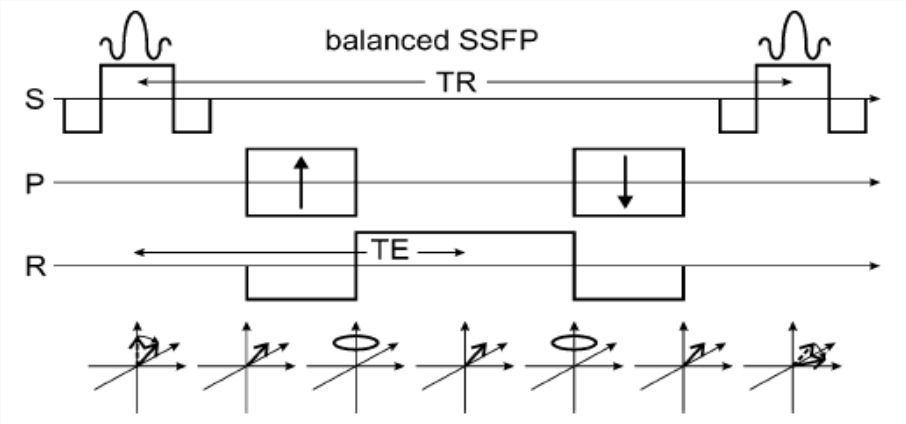
$$\min_{\mathbf{x}} \quad \|T\mathbf{x}\|_1$$
$$\text{subject to} \quad \|\mathbf{b} - A\mathbf{x}\| \leq \epsilon$$

- Synthesis Formulation

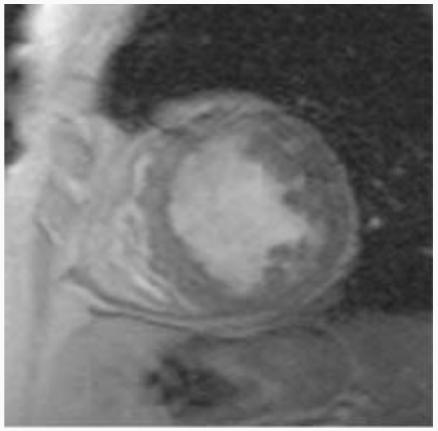
$$\min_{\theta} \quad \|\theta\|_1$$
$$\text{subject to} \quad \|\mathbf{b} - A\Psi\theta\| \leq \epsilon$$

Application : Cardiac MRI

- Balanced SSFP



Sequence diagram of b-SSFP



FLASH

b-SSFP

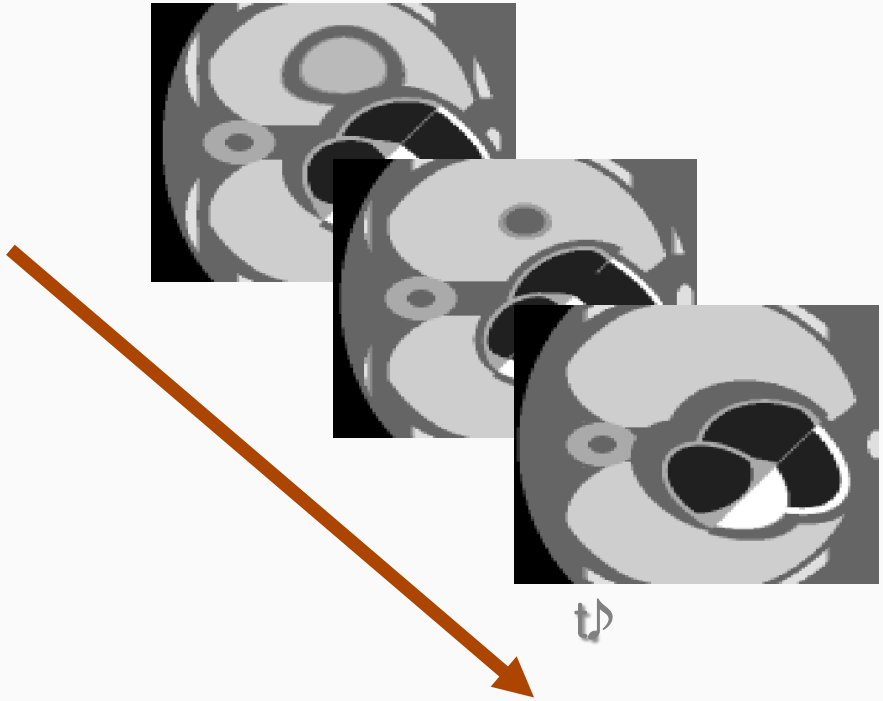
- **Demanding Acquisition Requirements**

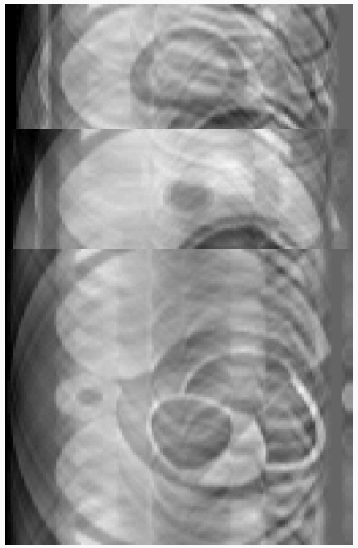
Matrix size	TR (msec)	Views per Segment	Temporal Resolution	Spatial Resolution	Total Acq. time/slice	Total Acq. time/volume
100x256	4	15	60 msec	$2.5mm \times 1.4mm$	7 sec	56 sec
196x256	4	15	60 msec	$1.3mm \times 1.4mm$	13 sec	104 sec

An example of Cartesian TrueFISP parameter for cardiac imaging. The acquisition time is calculated based on 60bpm patients with 8 slice acquisition. Flip angle of 40-60° is commonly used.

K. Scheffler et al, Eur Radiol (2003) 13:2409–2418

Aliasing in accelerated Dynamic MRI



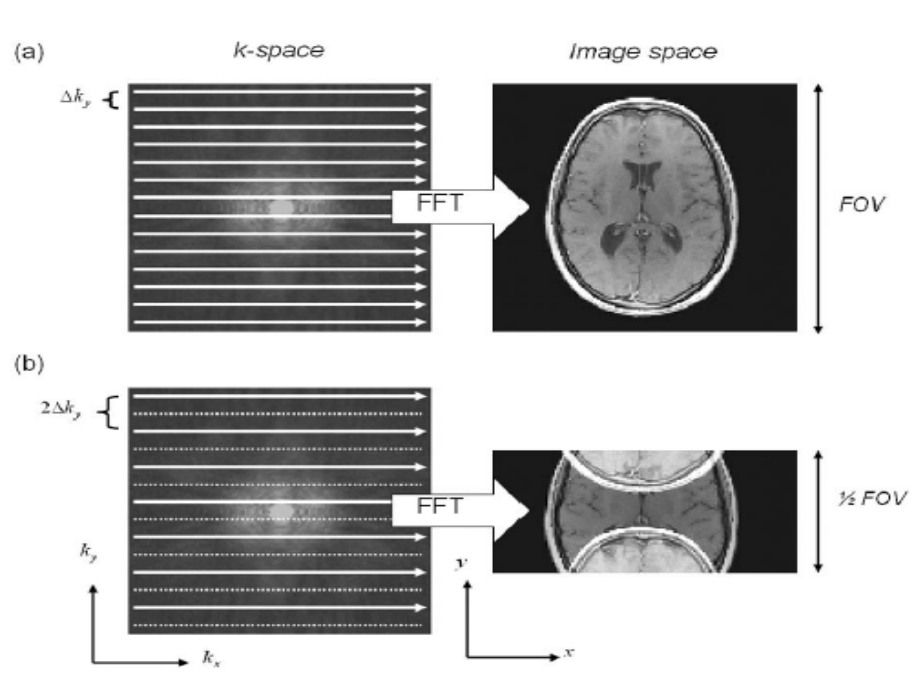


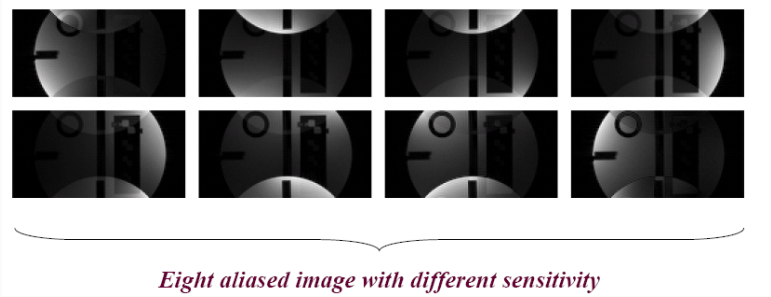
Down sampling along k-t space

→ Aliasing artifact

Classical Methods

- Parallel imaging (SENSE, GRAPPA)
  - Prussmann et al, 1999, Griswold et al 2002
- k-t space method
  - UNFOLD
  - UNFOLD-SMASH, TSENSE
  - K-t BLAST/SENSE
  - PARADISE, PARADIGM

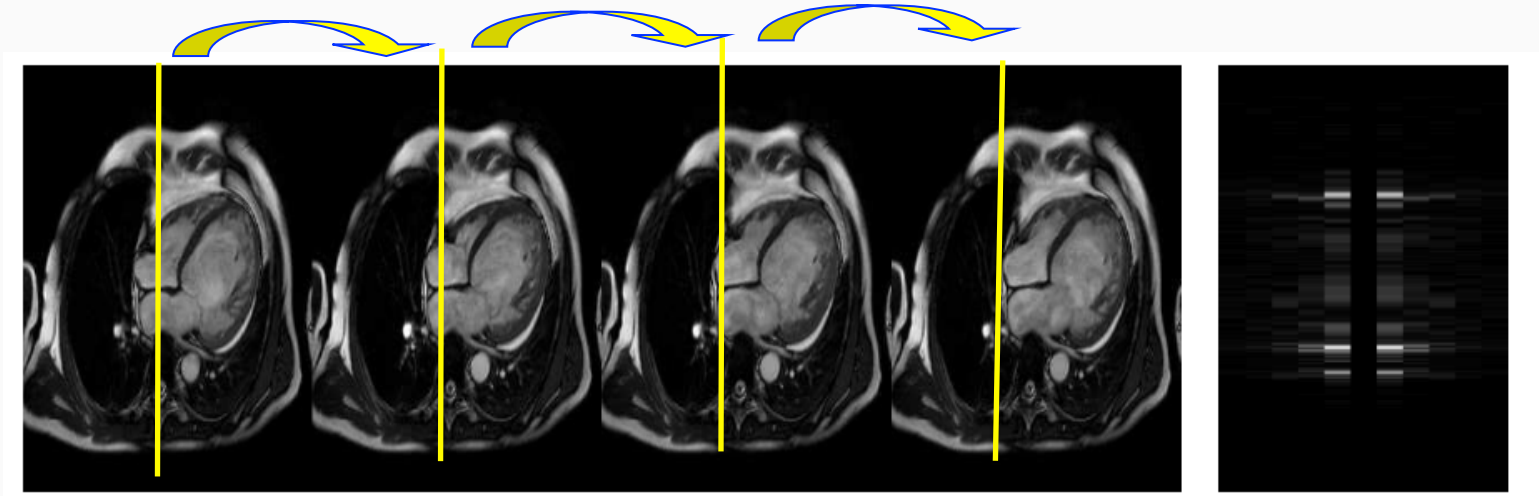




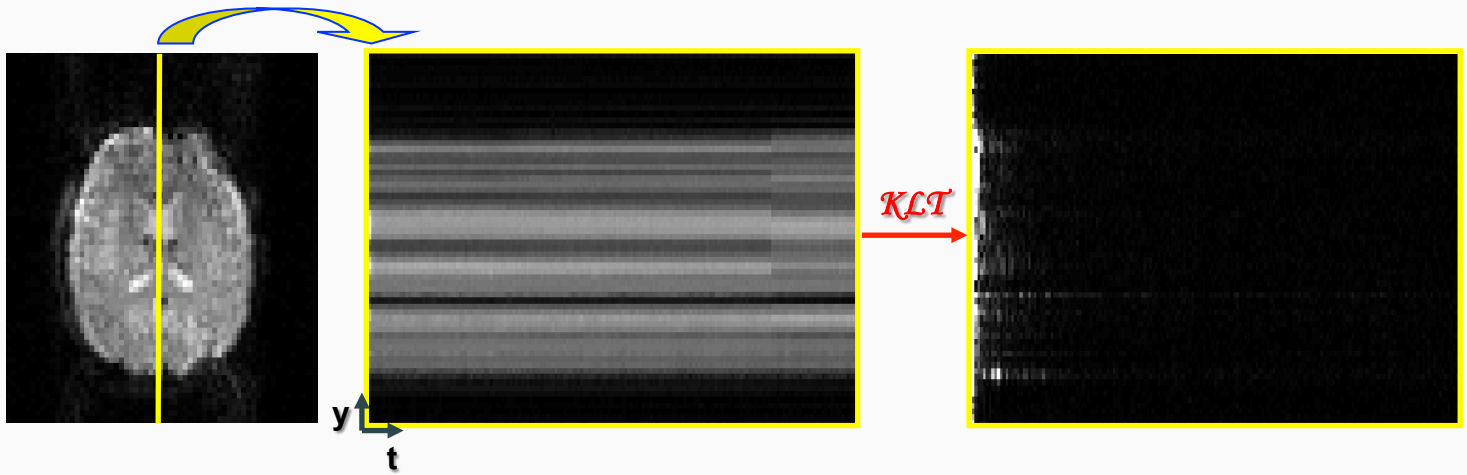
Blaimer et al, Top Magn Reson Imaging • V15, N 4, August 2004

How to Improve ? Sparsity !!

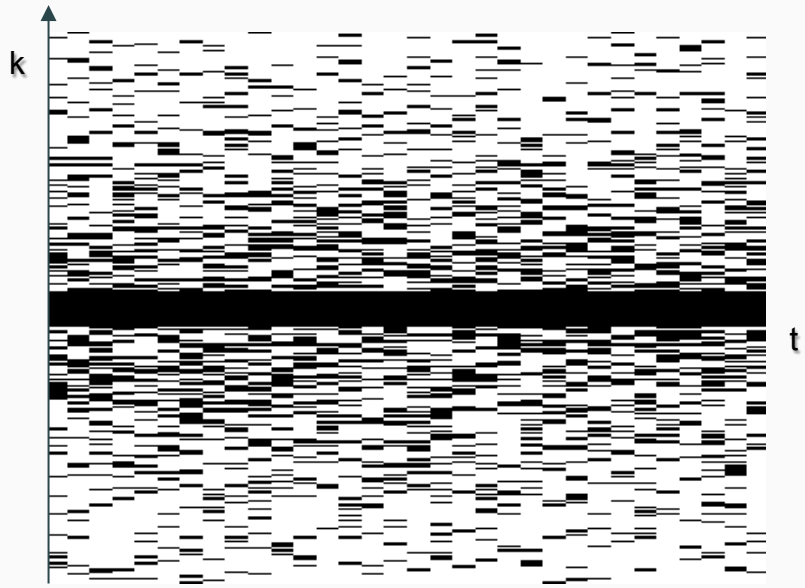
Cardiac MR



fMRI



RIP in Dynamic MR



Temporal transform

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \Phi\Psi\mathbf{x}$$

k-t space sample

Random phase encoding

k-t FOCUSS

(Jung et al, PMB:2007, MRM:2009(a), MRM:2009(b))

k-t FOCUSS (our method)

$$x_{n+1} = x_0 + \Theta_n A^H (A\Theta_n A^H + \lambda I)^{-1}(y - Ax_0)$$

$$\Theta_n = W_n^H W_n = diag\left(|x_{n,1}|^{2p}, \cdots, |x_{n,N}|^{2p}\right), \quad 0.5 \le p \le 1$$

Optimal from compressed sensing perspective ♪

When there is no update, k-t FOCUSS is exactly same with k-t BLAST/SENSE

k-t BLAST/SENSE (J. Tsao, MRM, 2003)

$$x = \bar{x} + \Theta_0 A^H (A\Theta_0 A^H + \lambda I)^{-1}(y - A\bar{x})$$

$$\Theta_0 = diag\left(|x_{n,1}|^{2}, \cdots, |x_{n,N}|^{2}\right)$$

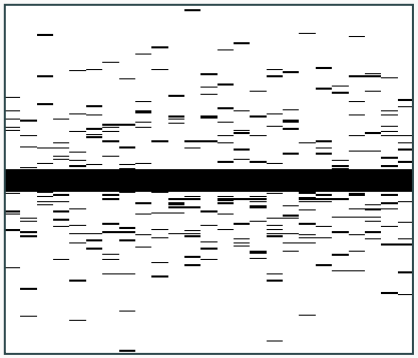
No-update♪

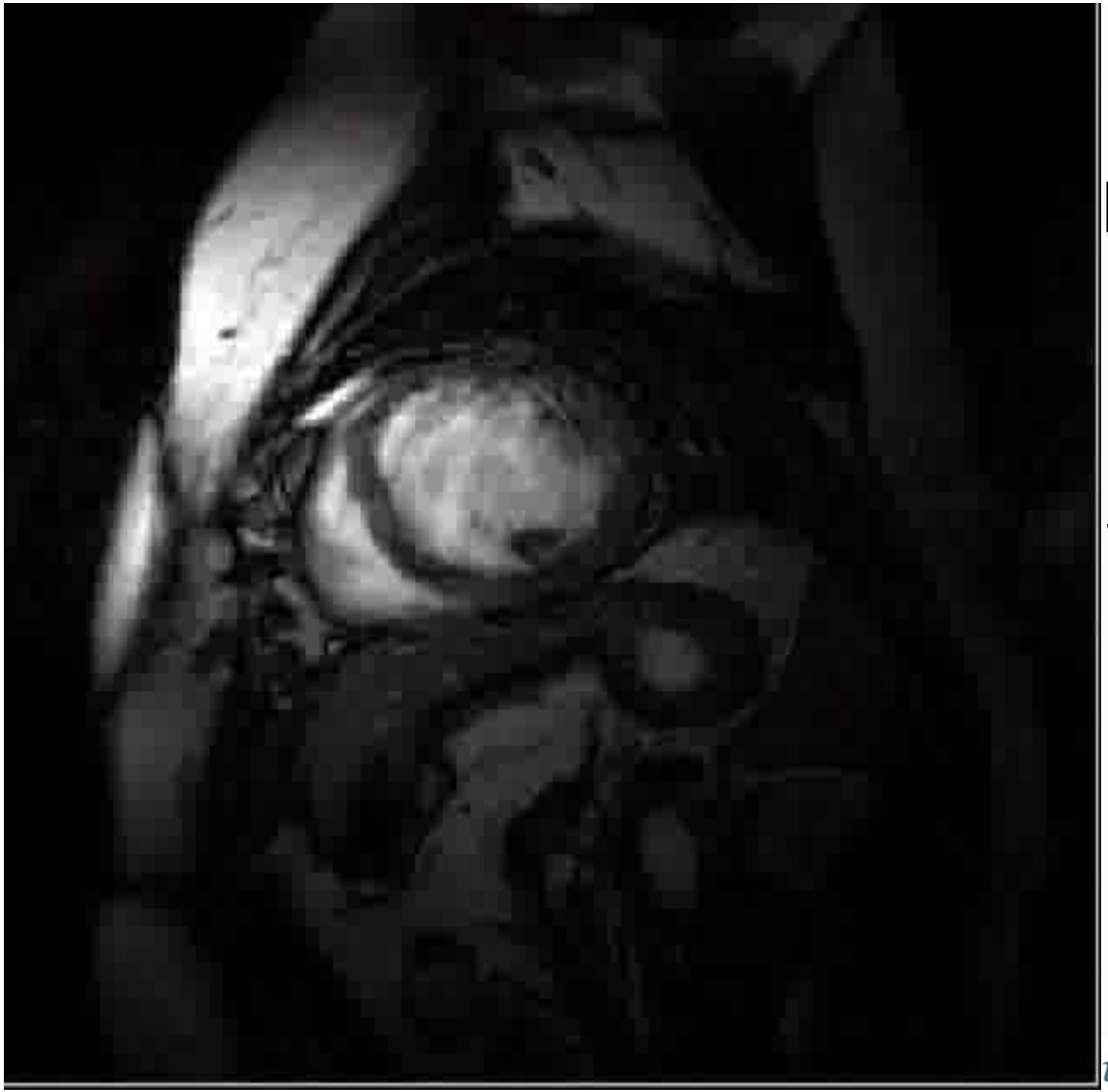
Does not solve L1 minimization problem ♪ ➡ Not Optimal from CS perspective

11 x accelerated

Zero-padding inverse FFT from measurements

Sampling pattern

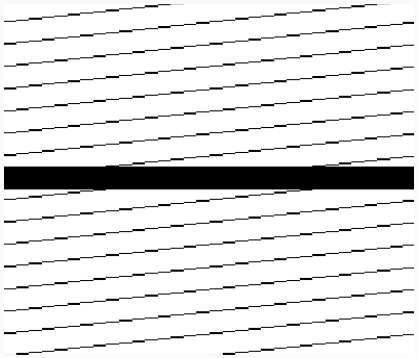




11 x accelerated

k-t BLAST

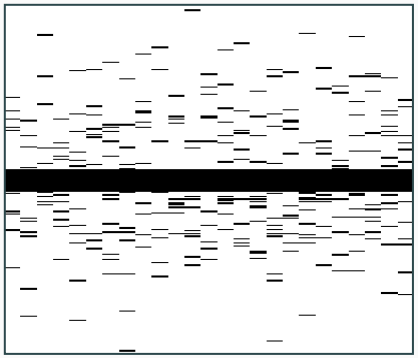
Sampling pattern



11 x accelerated

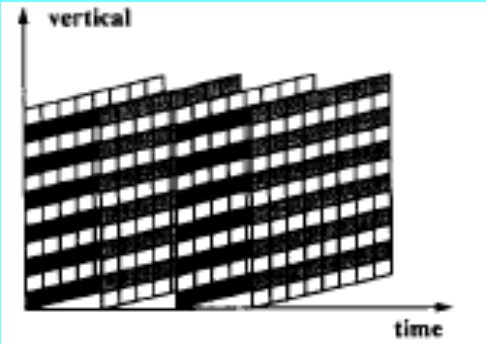
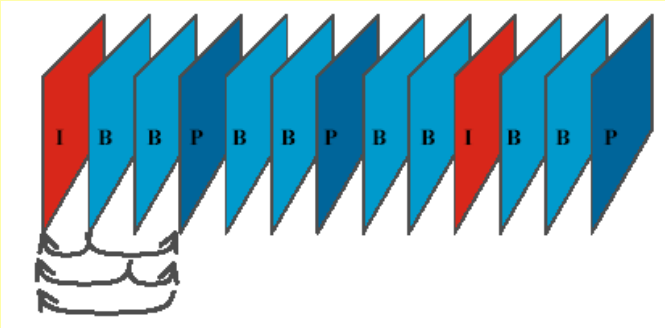
k-t FOCUSS with temporal average

Sampling pattern



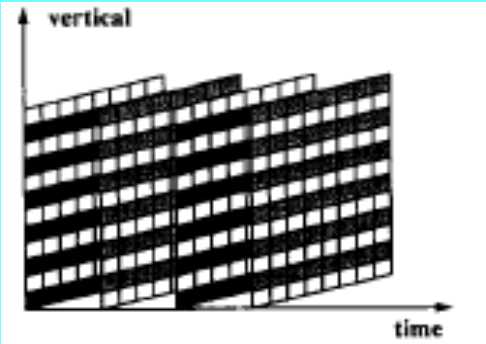
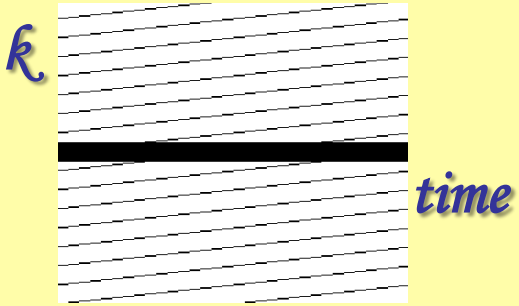
MOTION-COMPENSATED
COMPRESSED SENSING

Lessons from HDTV History

MUSE	MPEG
The first HDTV (Japan) **(obsolete)**	**Modern Standard** **(MPEG1,2,4..)**
quincunx sampling  *Figures from Kovacevic et al, IEEE TIP, 1993*	**ME/MC** 
Lattice sampling theory **→ Low compression**	**ME/MC + residual coding** **→ High compression**

* MUSE: MUltiple Sub-nyquist Encoding systems

MUSE vs k-t MR

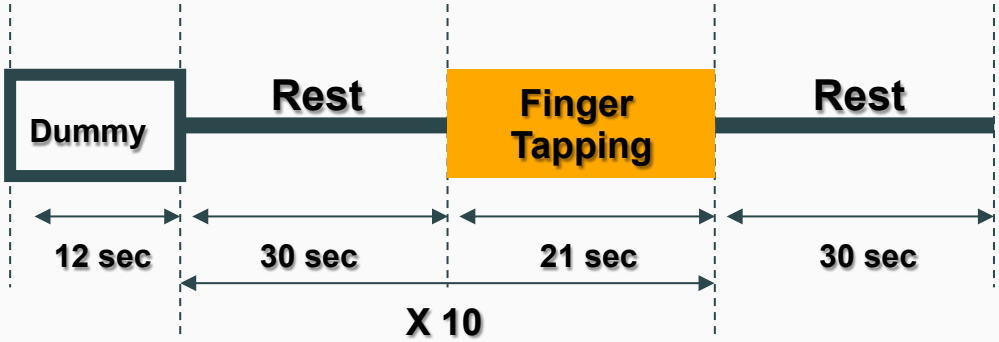
MUSE	k-t MR
The first HDTV (Japan) **(obsolete)**	**UNFOLD** **k-t BLAST/SENSE**
quincunx sampling  *Figures from Kovacevic et al, IEEE TIP, 1993*	k-t space downsampling 
Lattice sampling theory → Low compression	Lattice sampling theory → Low acceleration !!

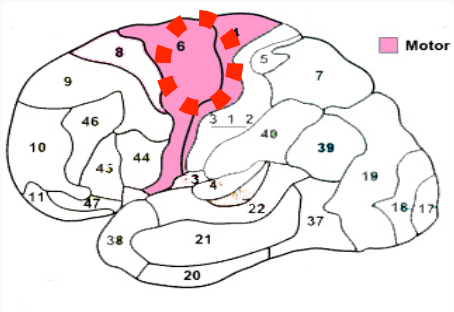
*** MUSE: MUltiple Sub-nyquist Encoding systems**

Reference is often Free

- Cardiac MR
  - During volume acquisition, diastole phase can be acquired

- fMRI



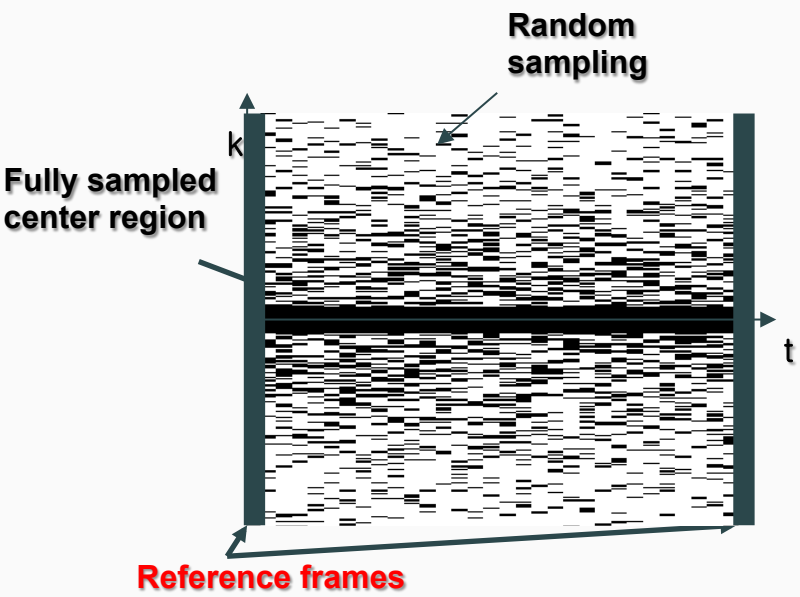


- MR angiography, vocal tract, etc.
- example) RIGR, SPEAR, etc.

Random Sampling with Reference

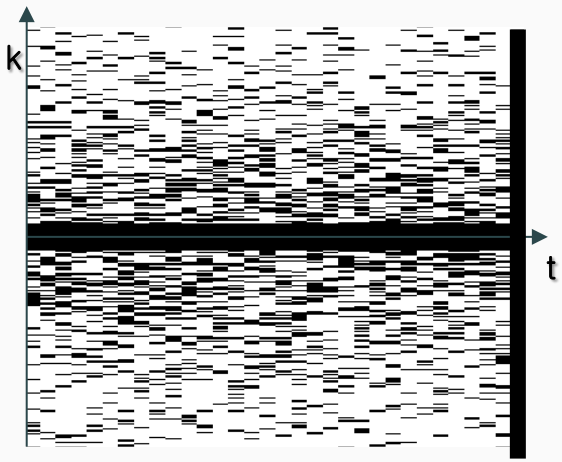
We want to distribute k-space samples to allow
prediction and residual coding

When two reference frames can be obtained,

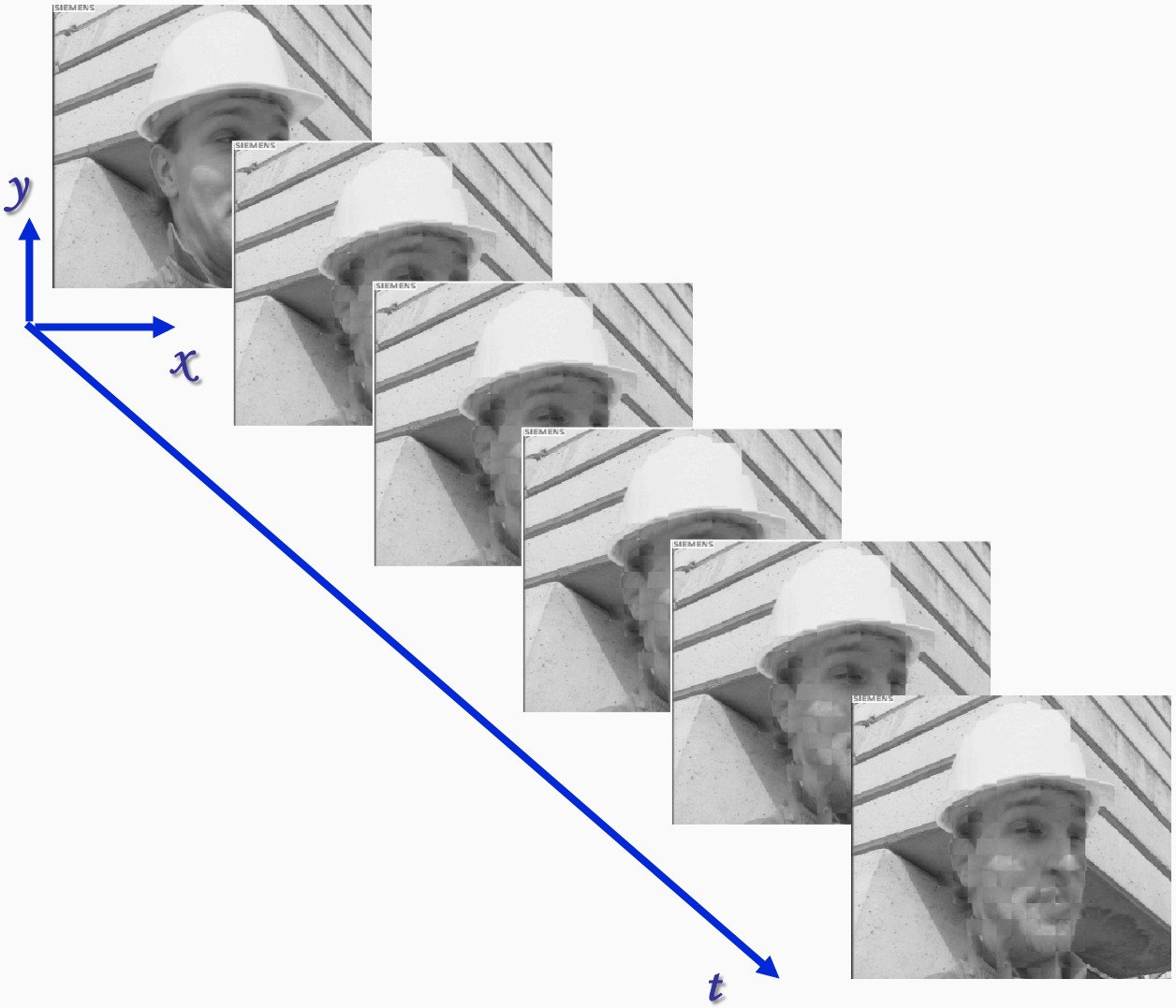


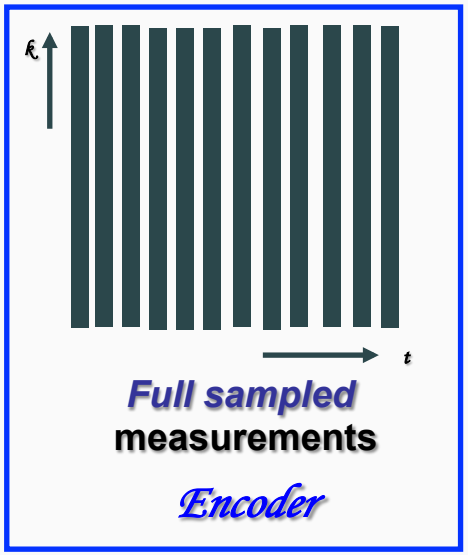
When one reference frames can be obtained

x_0

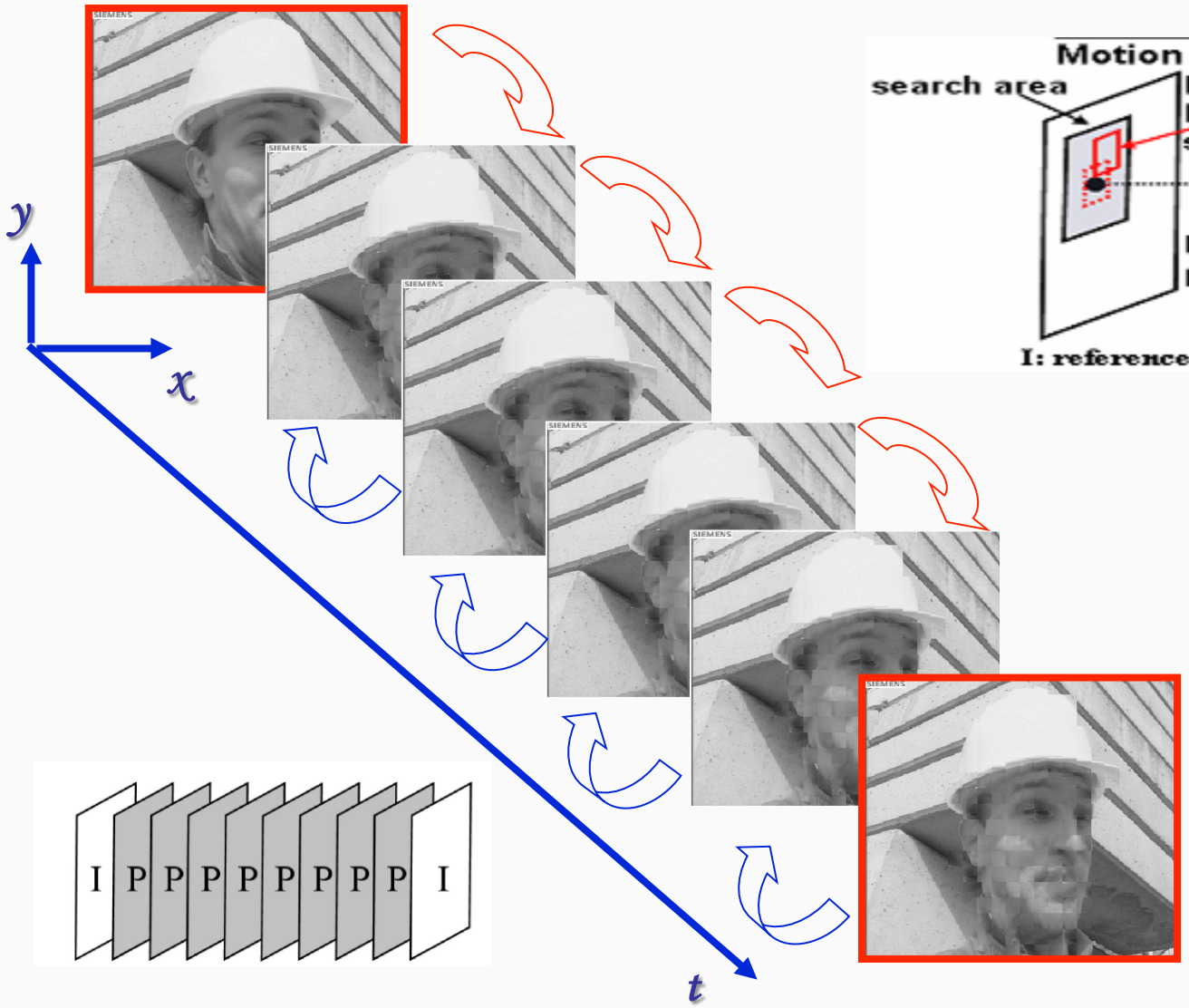


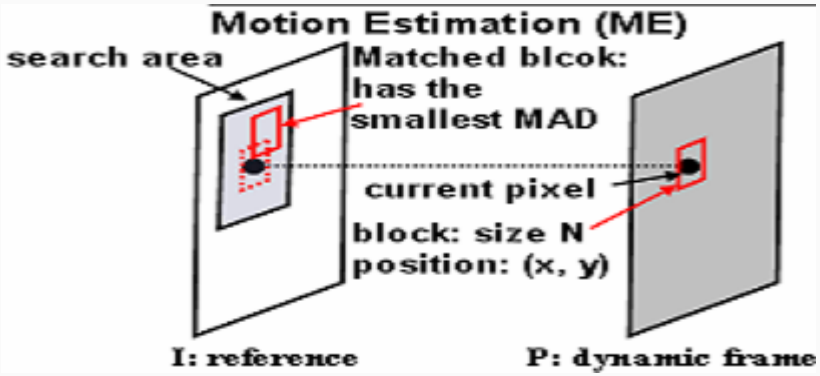
ME/MC in MPEG Video

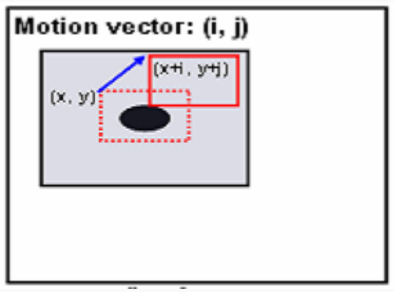




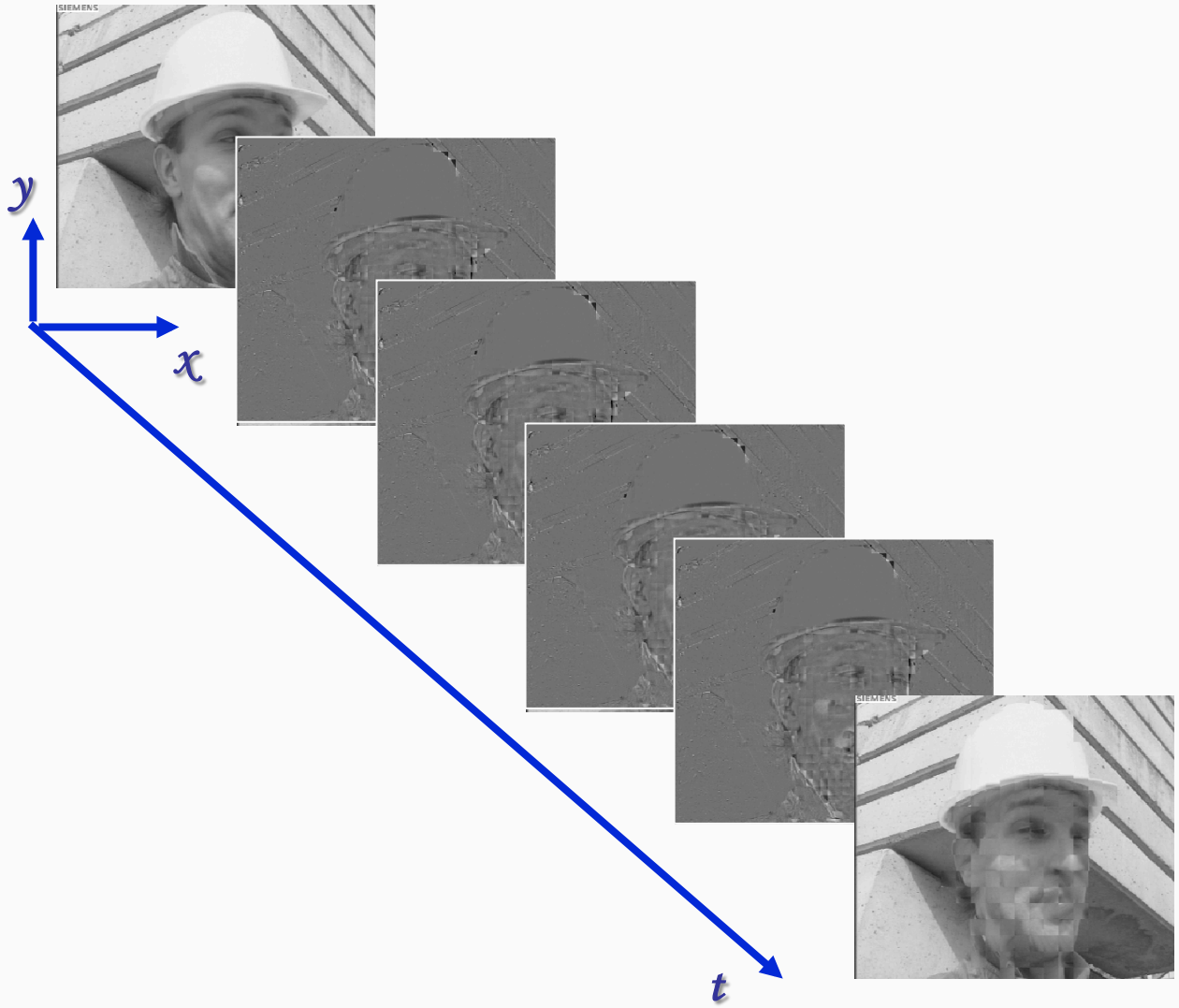
ME/MC in MPEG Video







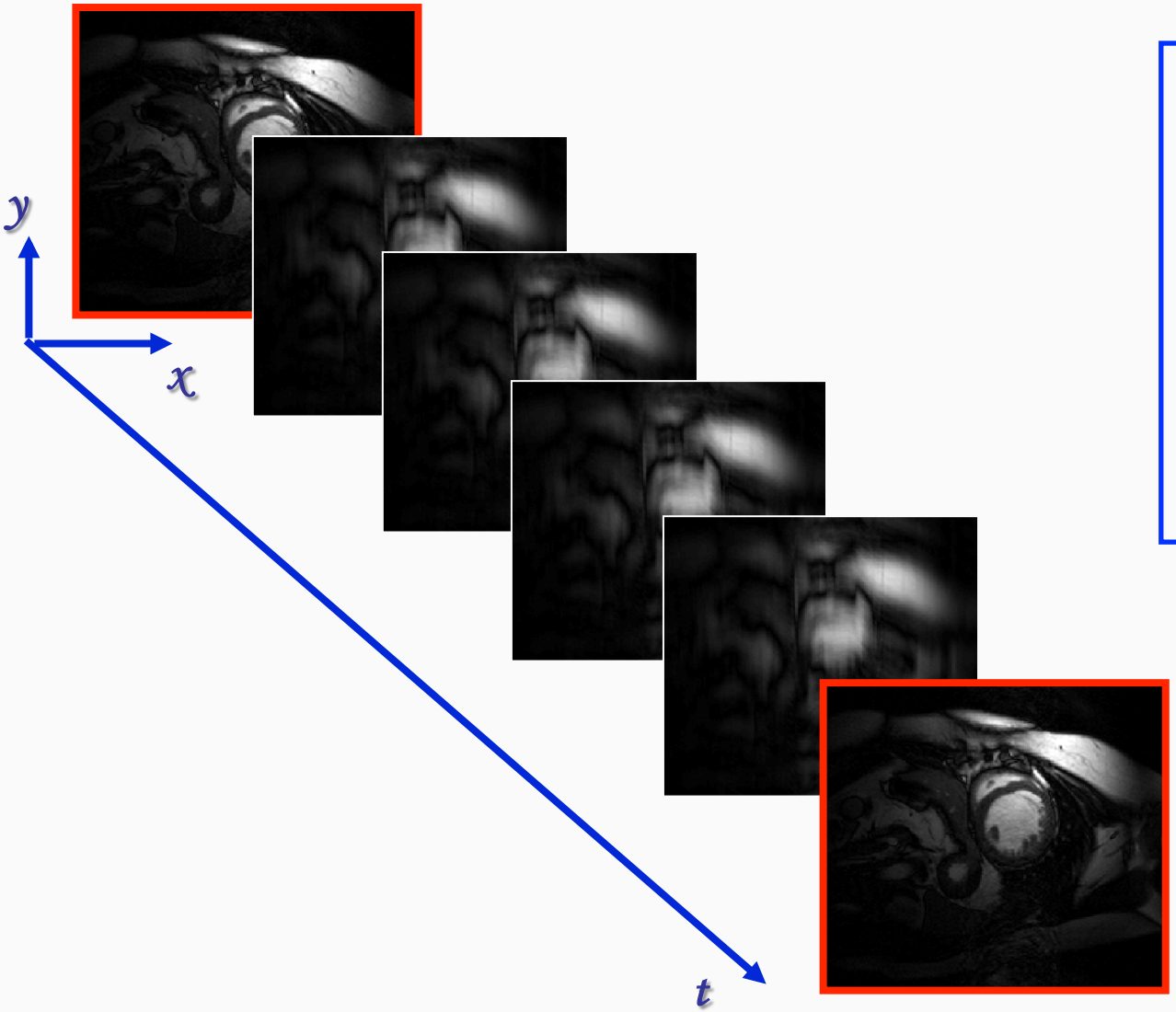
ME/MC in MPEG Video

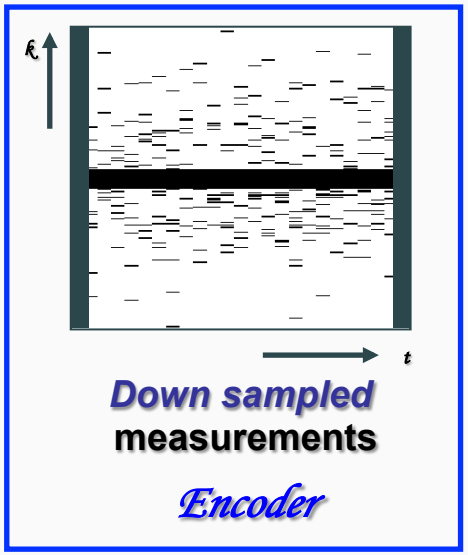


Residual encoding

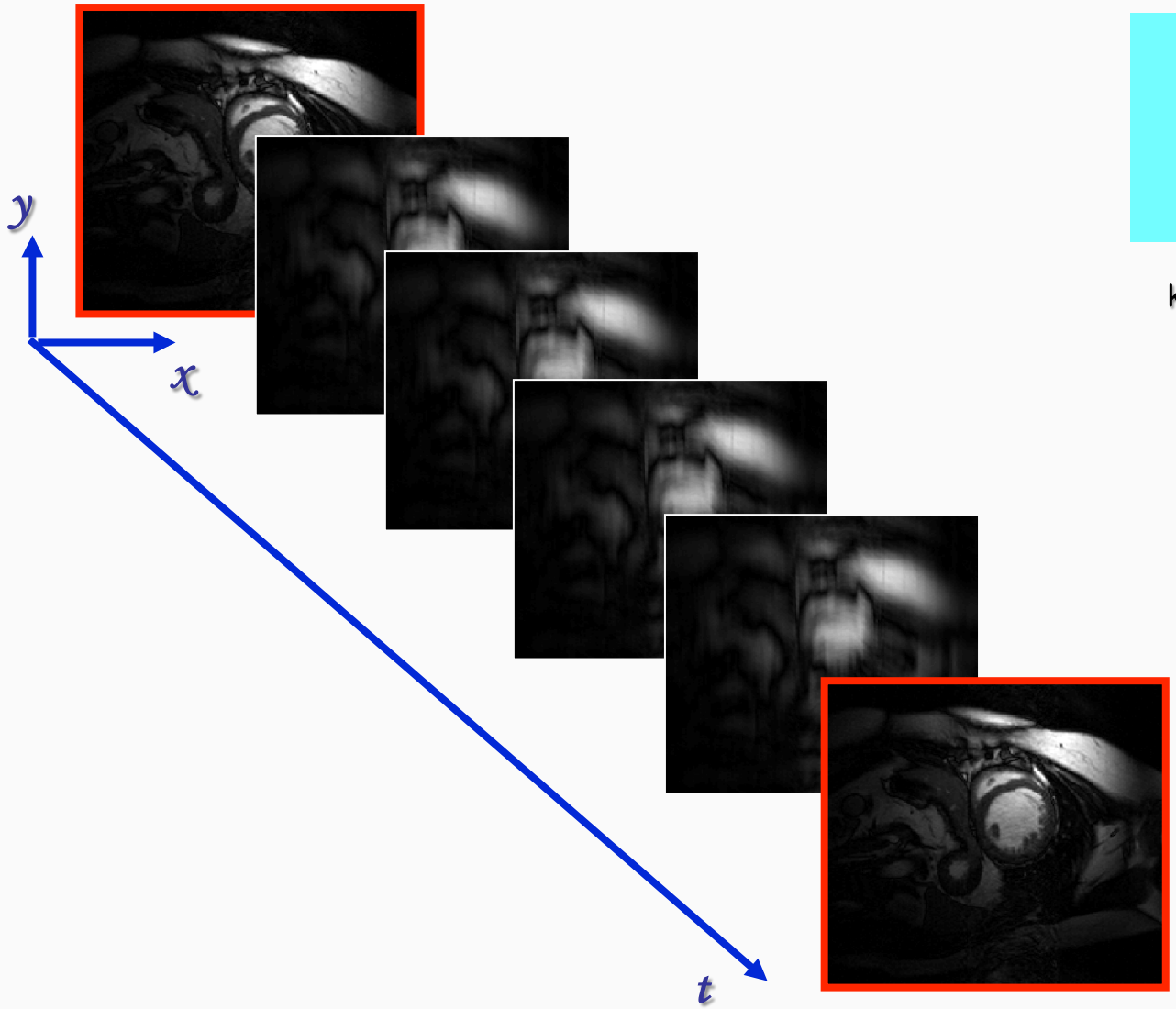
ME/MC in Dynamic MRI

Refinable Motion Estimation

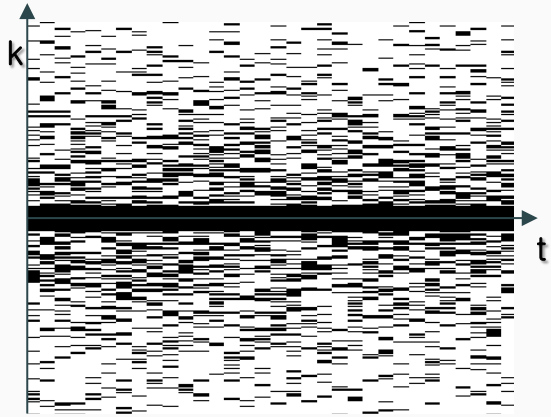




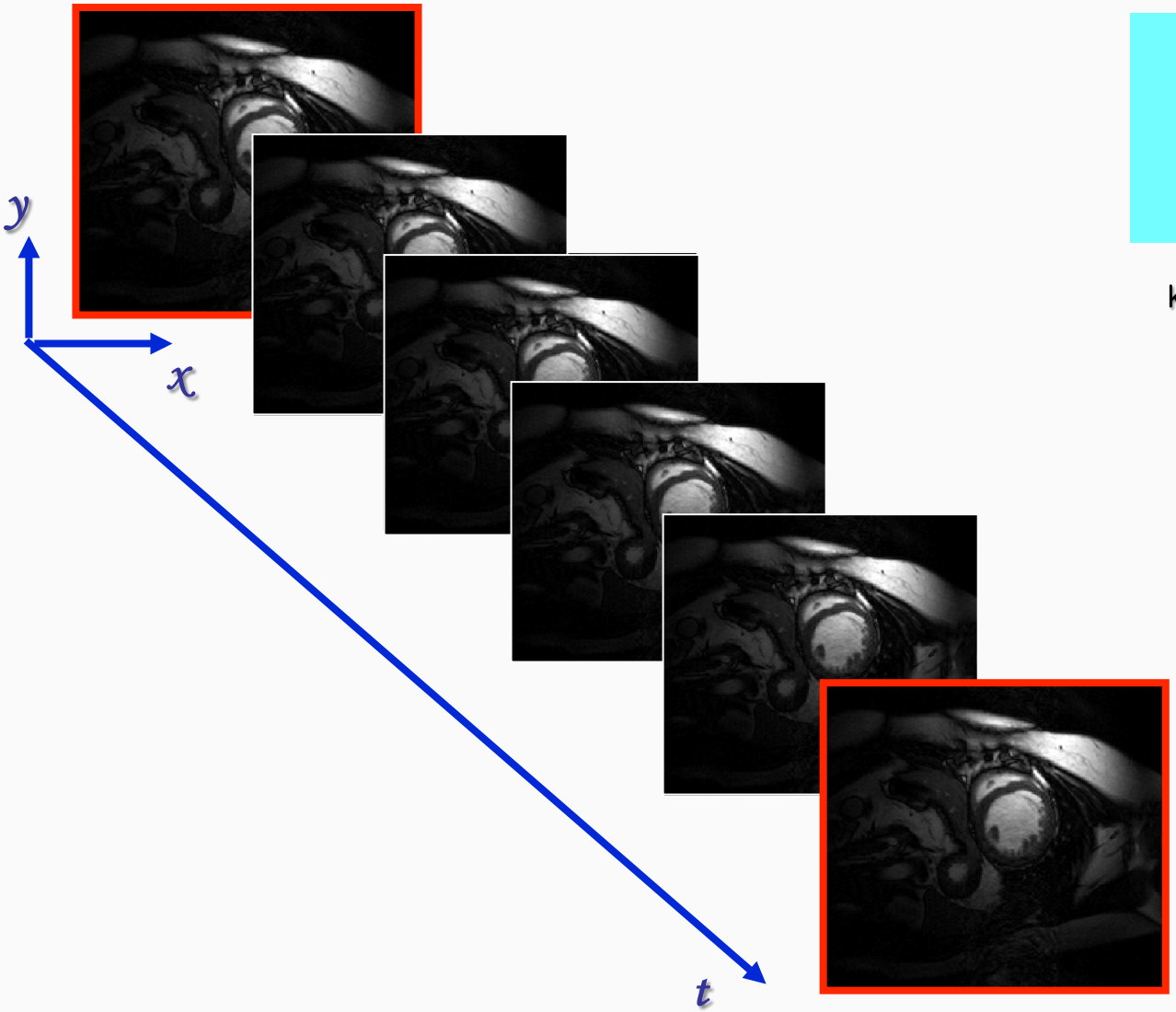
Refinable Motion Estimation



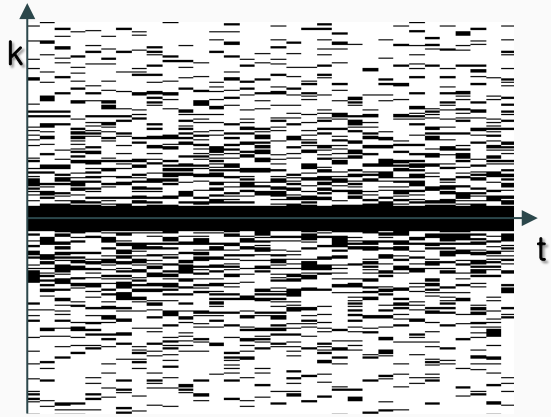
Initialization with k-t FOCUSS



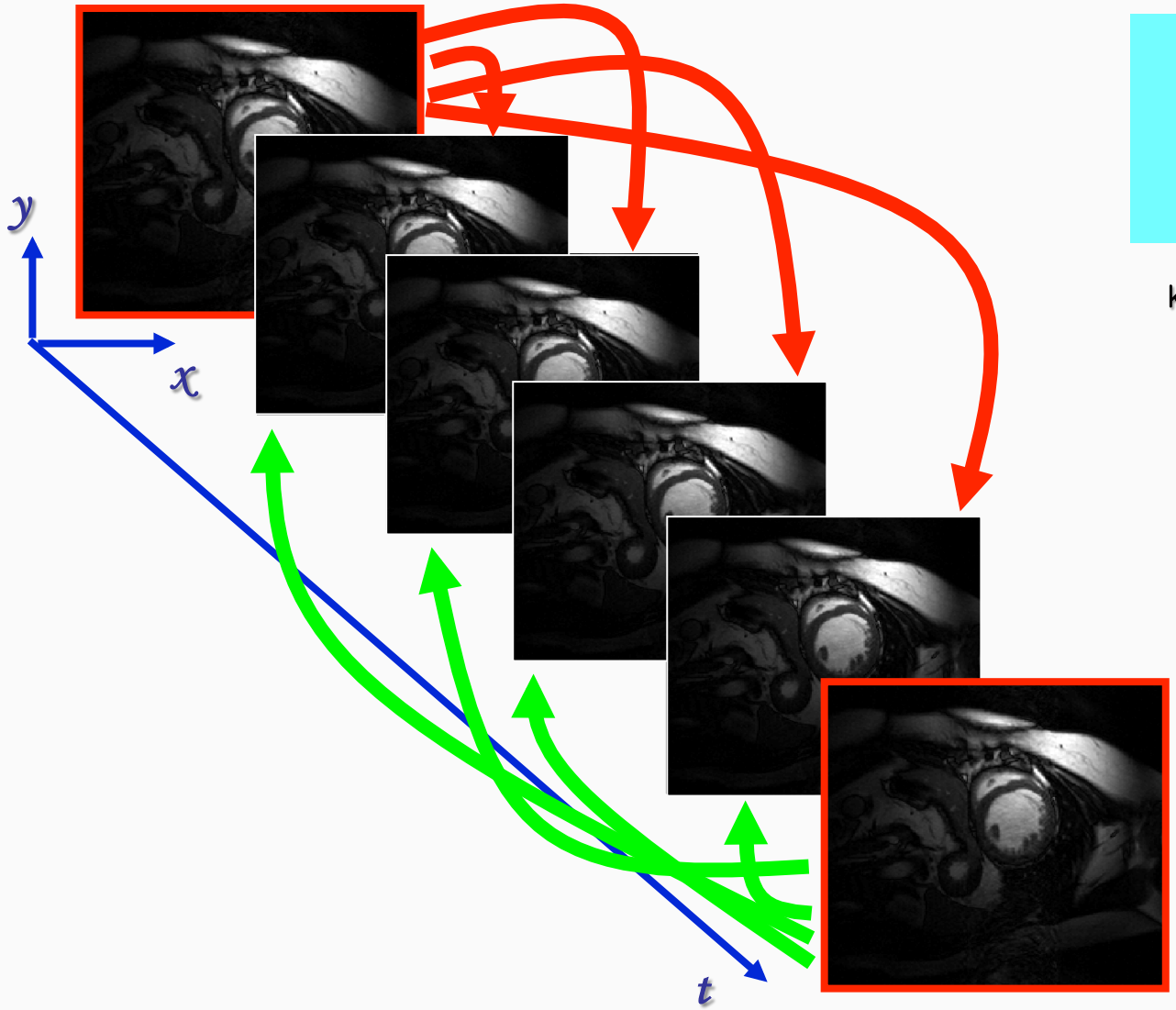
Refinable Motion Estimation



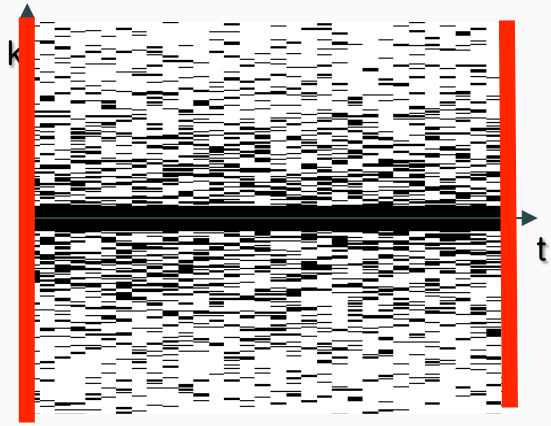
Initialization with k-t FOCUSS



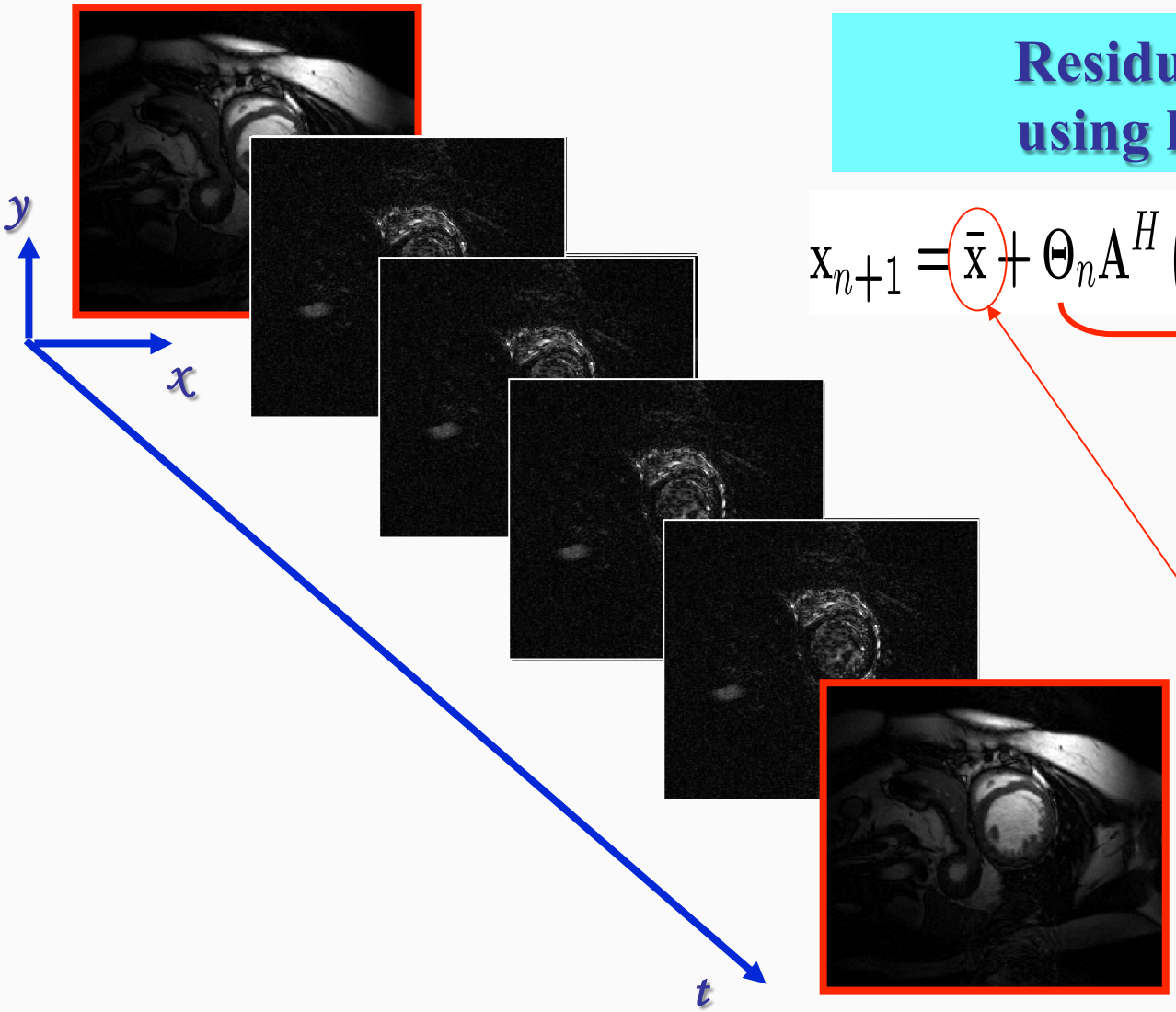
Refinable Motion Estimation



Apply ME/MC



Refinable Motion Estimation



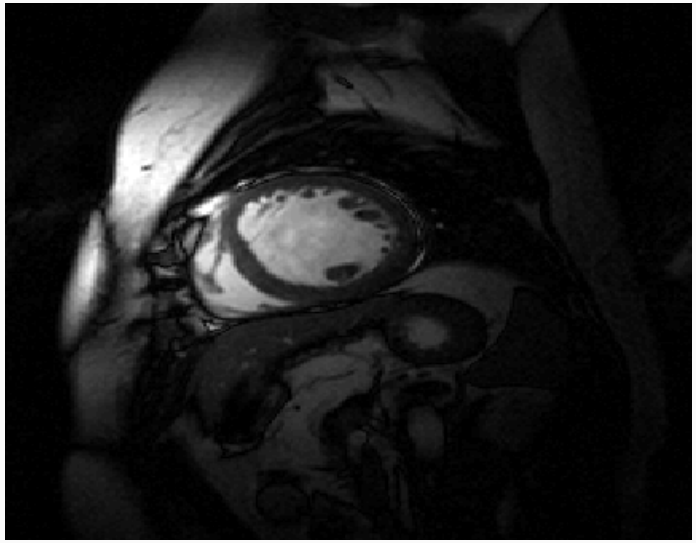
Residual Encoding using k-t FOCUSS

$$x_{n+1} = \bar{x} + \underbrace{\Theta_n A^H \left(A\Theta_n A^H + \lambda I\right)^{-1} (y - A\bar{x})}_{\text{Residual Encoding}}$$

ME/MC

Cine MRI

- Acquisition parameters
  - 1.5 T Philips scanner at Yonsei Unversity Medical Center in Korea
  - bSSFP
  - FOV = 345.00 x 270.00 mm^2
  - Matrix size : 256 x 220 (Read-Out x PE)
  - TR = 3.45ms
  - flip angle : 50°
  - heart rate : 66 bpm
  - # of cardiac phases : 25 frames

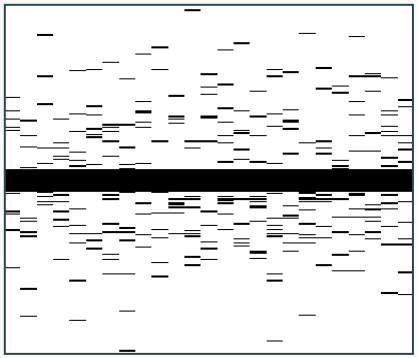


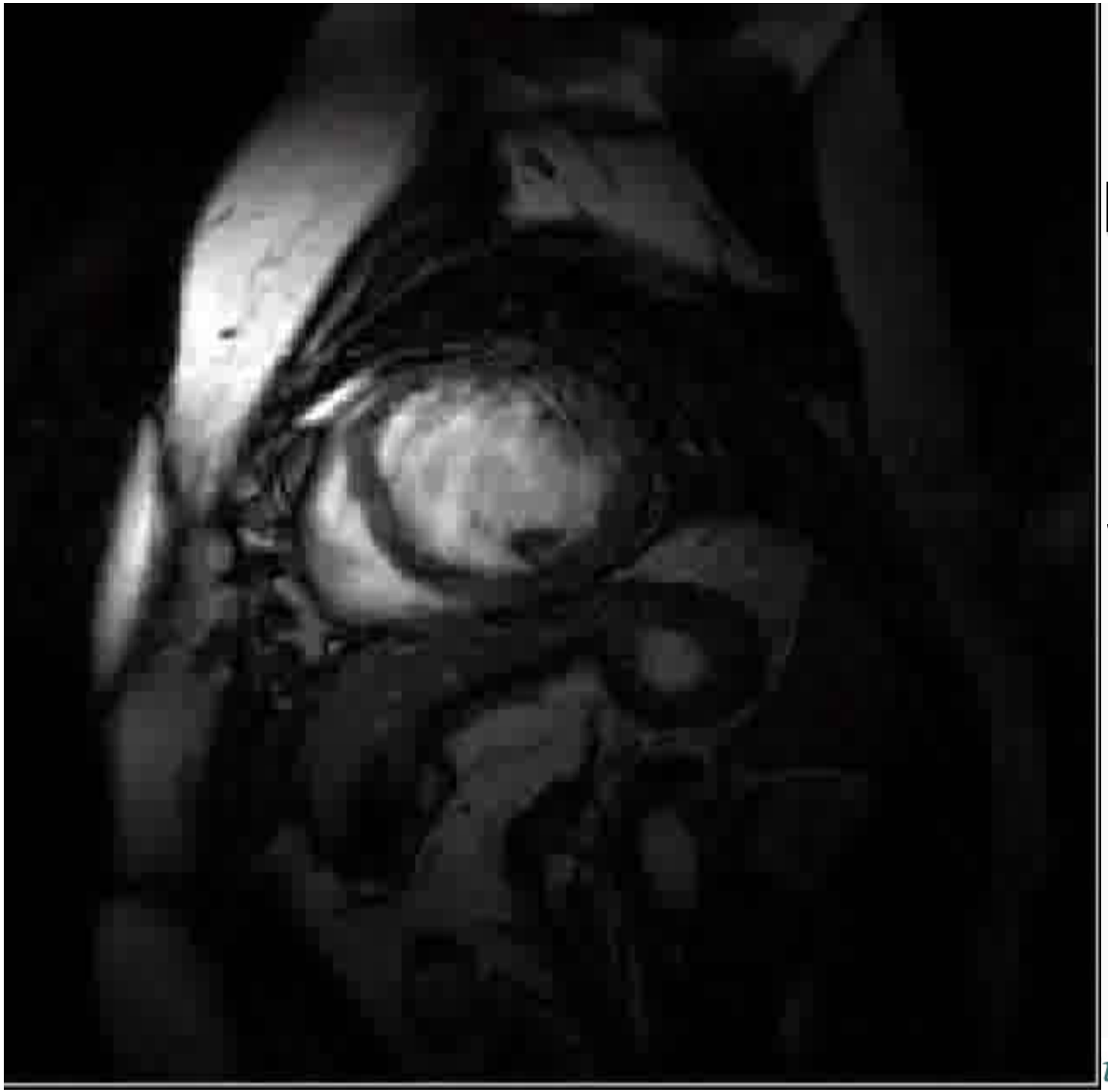
- Among fully sampled k-data, partial k-data were arbitrarily chosen.

11 x accelerated

Zero-padding inverse FFT from measurements

Sampling pattern

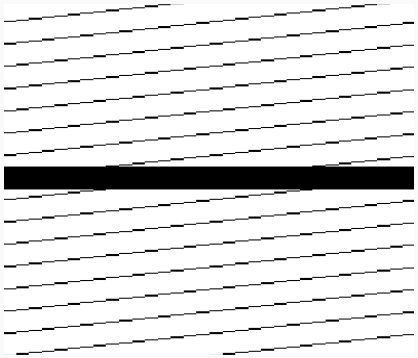


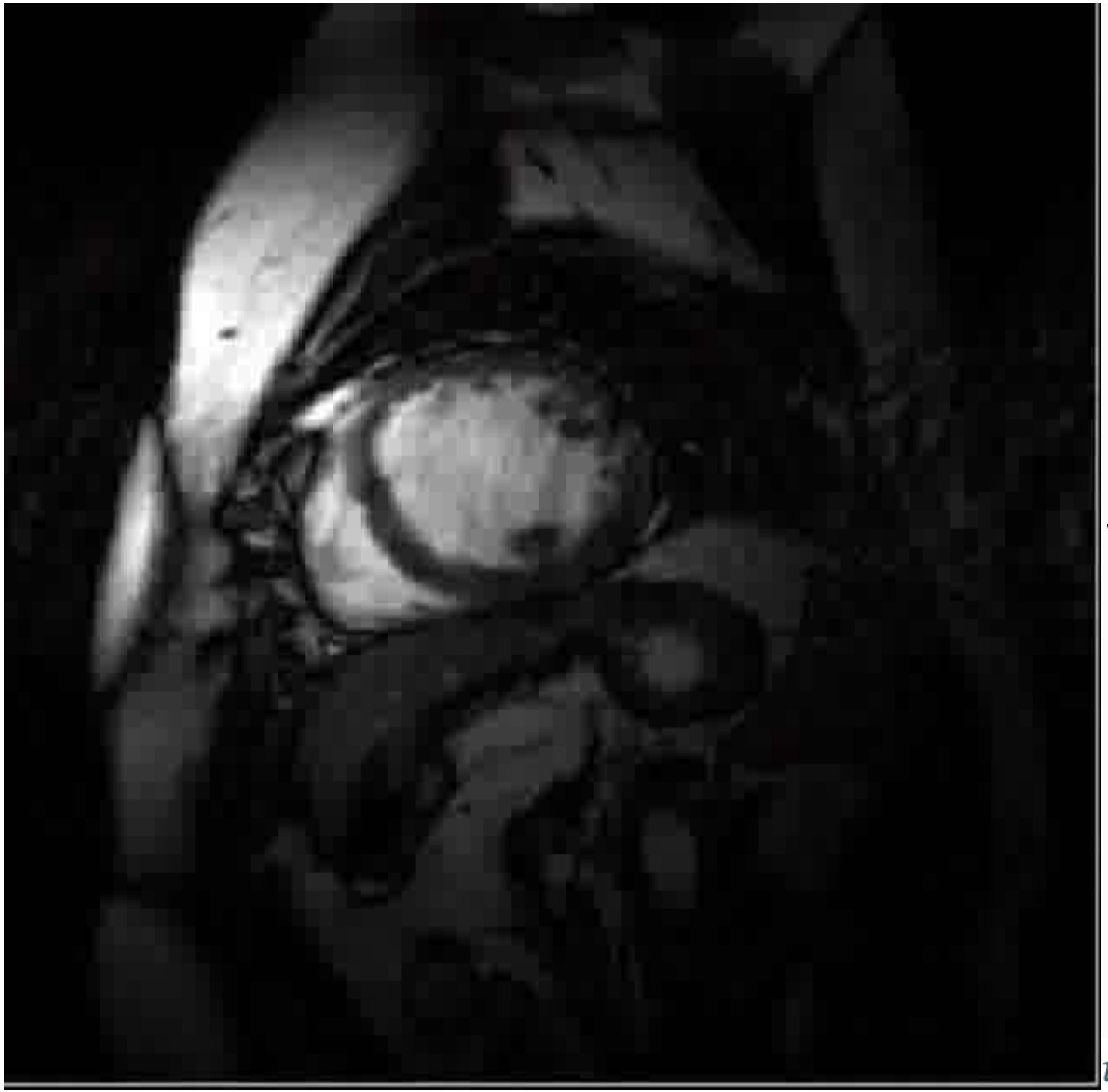


11 x accelerated

k-t BLAST

Sampling pattern

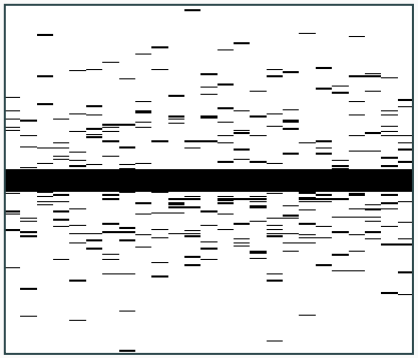


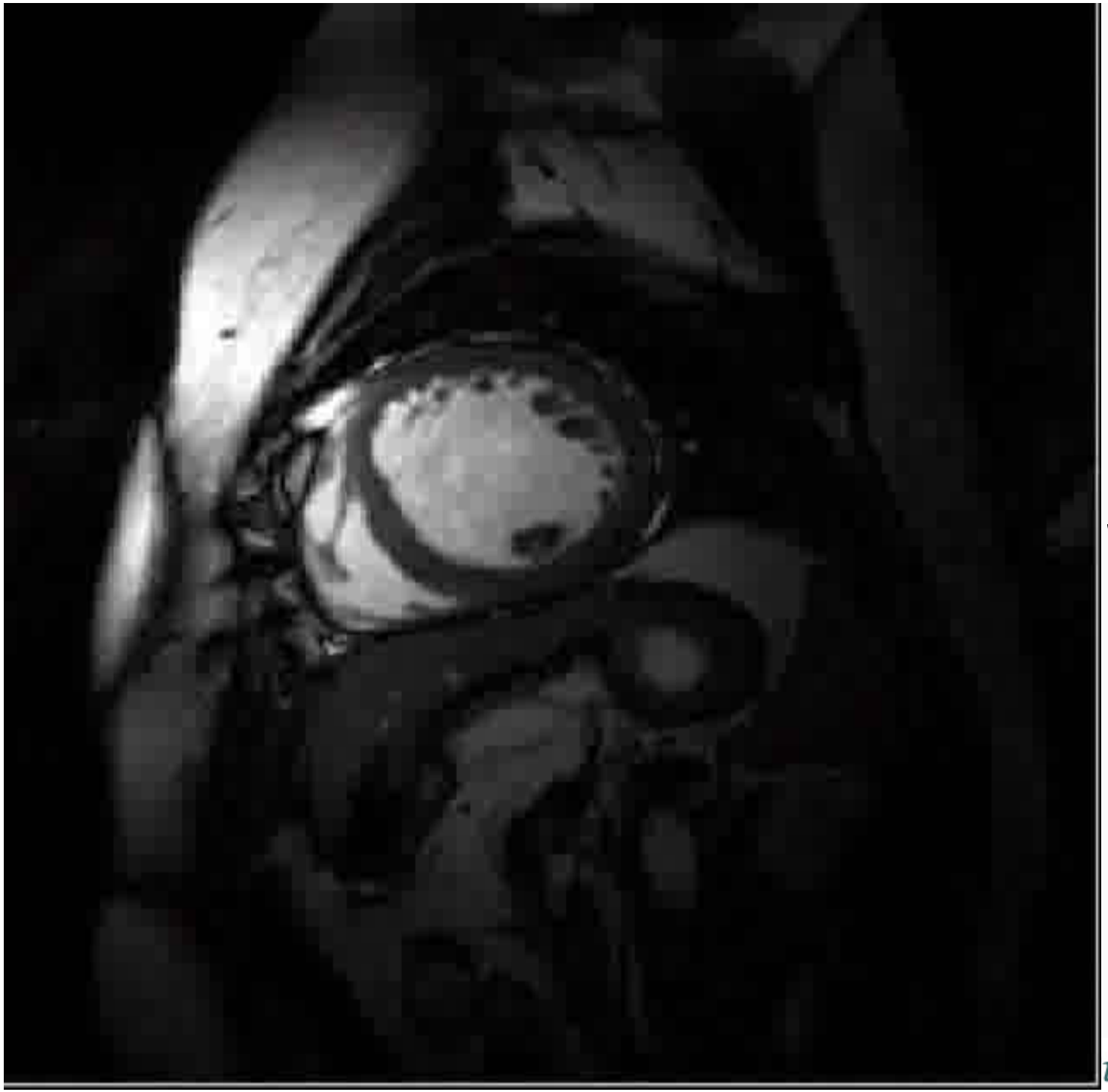


11 x accelerated

k-t FOCUSS

Sampling pattern



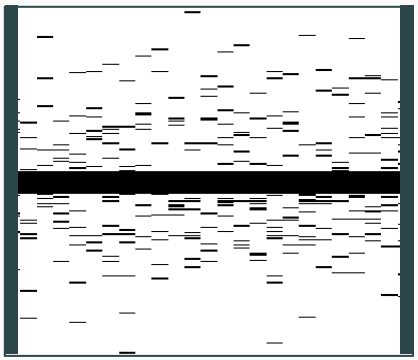


11 x accelerated

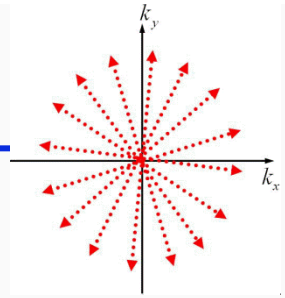
Motion Compensated

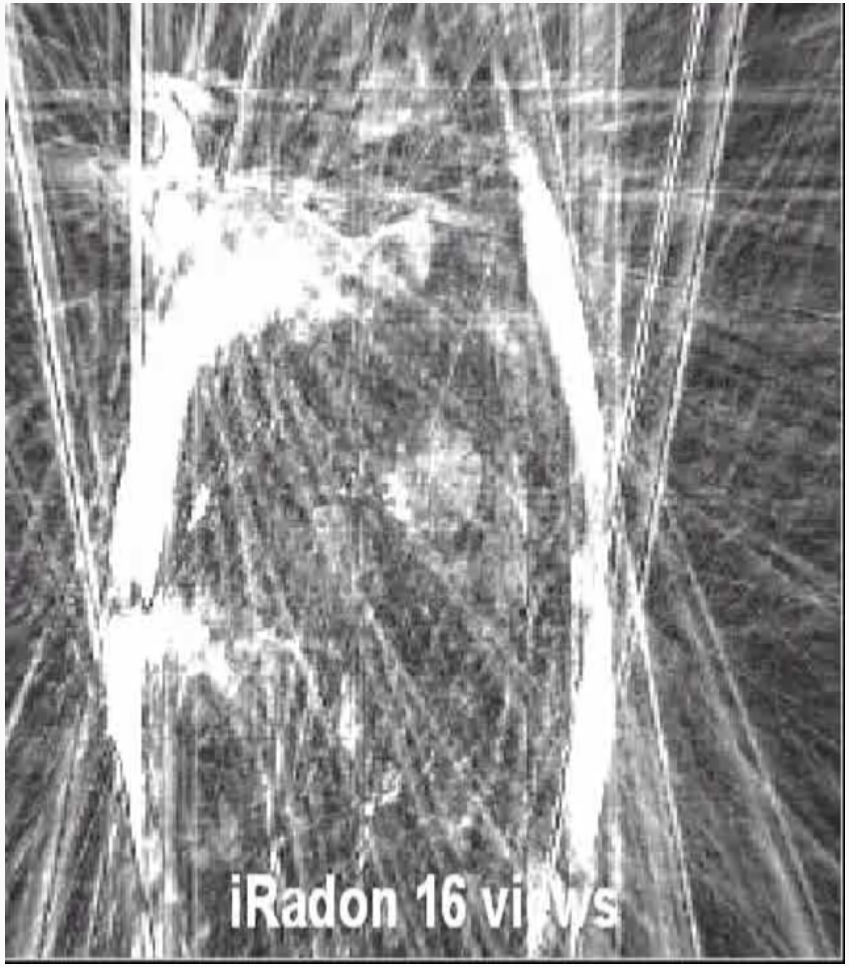
k-t FOCUSS

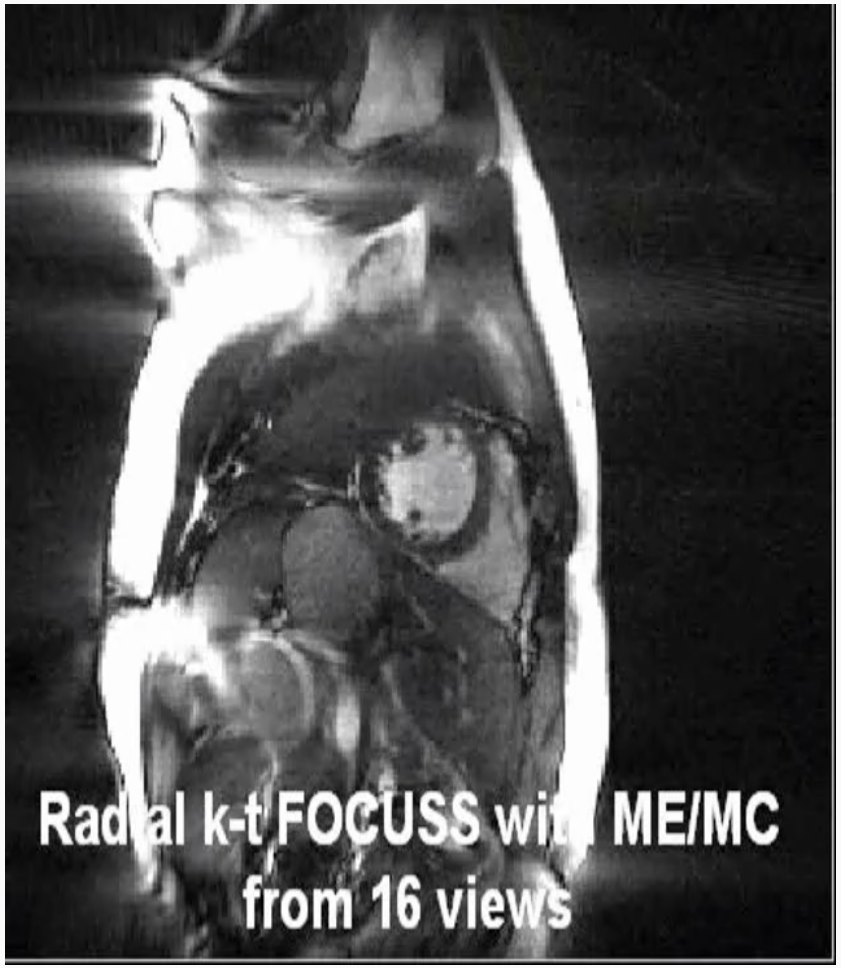
Sampling pattern



Radial Acquisition





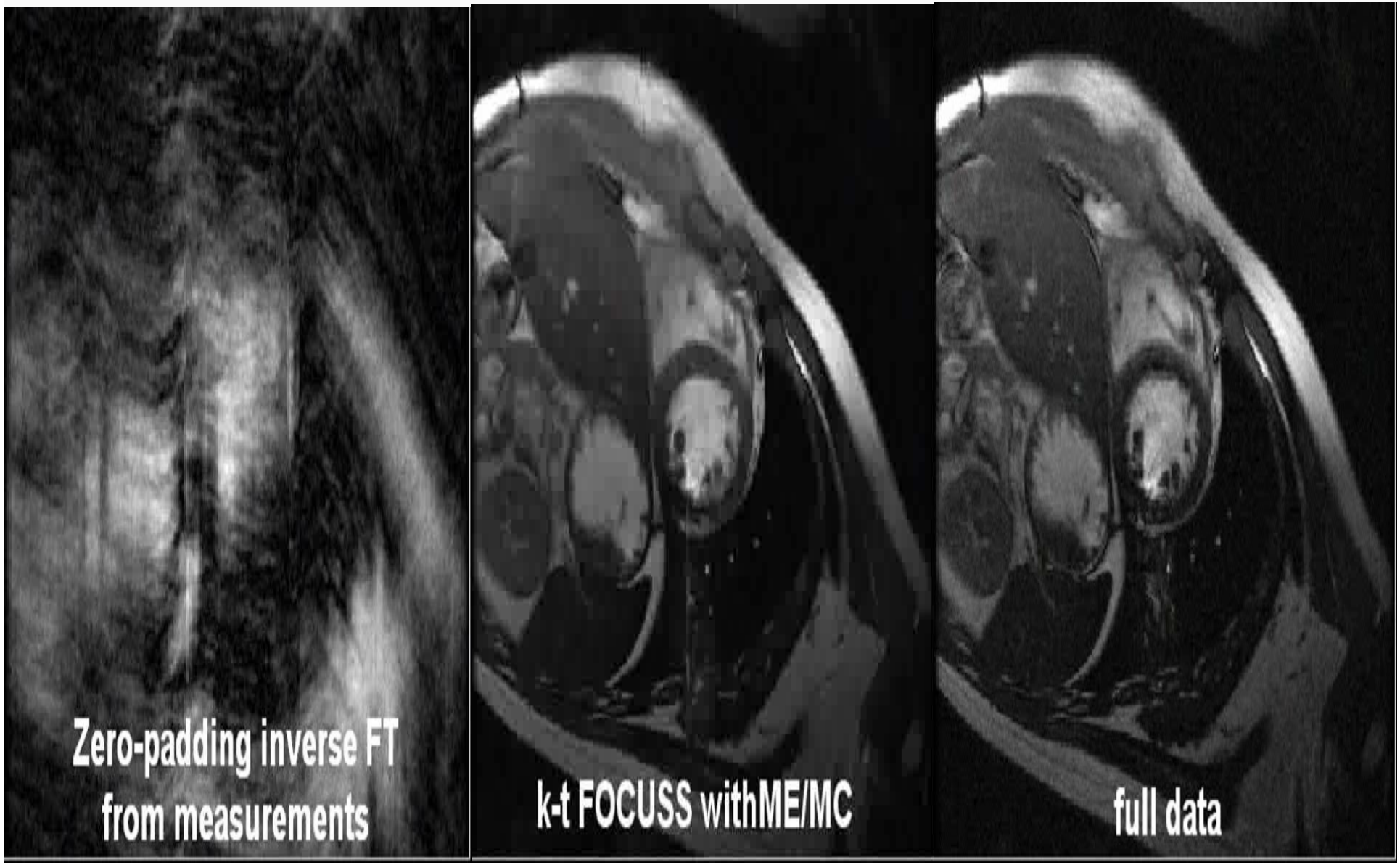


World-first In vivo Experiment

(Joint work with K. Nayak at USC)

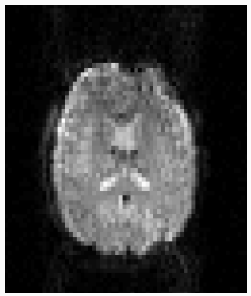
- Acquisition parameters
  - 3.0 T GE Signa EXCITE at the University of Southern California
  - bSSFP
  - FOV = 300 x 300 mm^2
  - Matrix size : 240 x 256 (PE x Read-Out)
  - TR = 3.7ms
  - views per segment = 10
  - flip angle : 45°
  - heart rate : 65 bpm
  - # of ardiac phases : 25 frames
  - acquisition time for 6 x accelerated data = 4 heart beats
  - acquisition time for fully sampled data = 24 heart beats

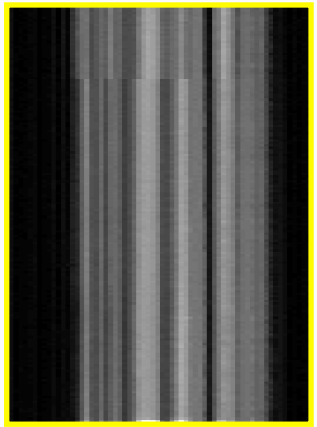
In Vivo Experiment (6x accel.)



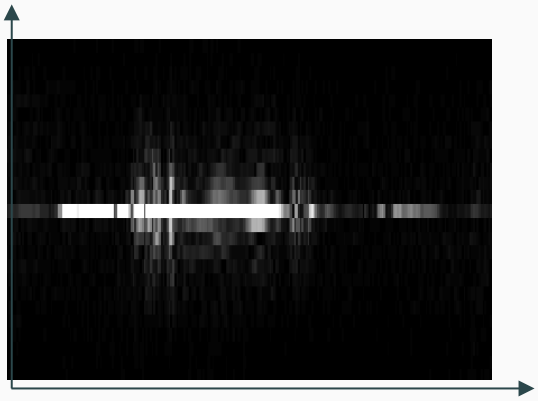
LEARNING-BASED
COMPRESSED SENSING

Which One Is Optimal ?

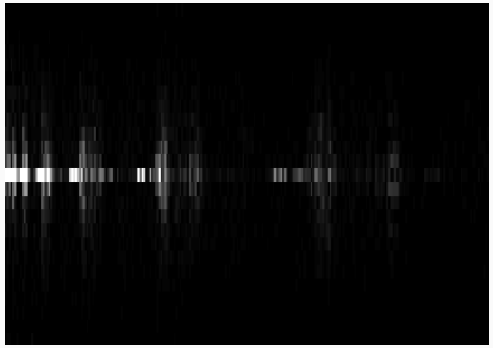




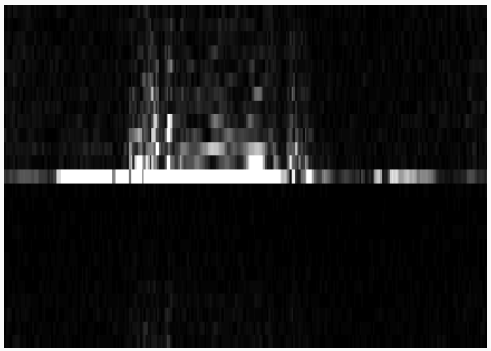
FT along t-axis



PCA along t-axis

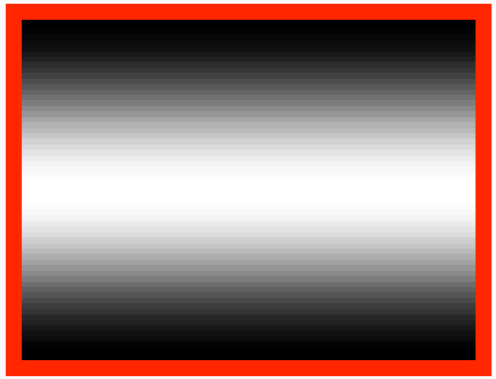


WVT along x-axis
+ FT along t-axis



KLT (PCA): optimal transform

Karhunen Loeve Transform is well known for an optimal sparsifying transform of general signals.



X

Considering SVD

$$X = U \Sigma V^T$$

KLT basis:

Matrix of Eigen vectors for XX^T

$$\Sigma = \begin{pmatrix} L & & 0 \\ & s & \\ 0 & & s \end{pmatrix}$$

L: Large Singular values

s: small Singular values

$$L >> s \rightarrow 0$$

Hence, KLT $U^T X = \Sigma V^T$ →

is SPARSE

PCA in k-t FOCUSS

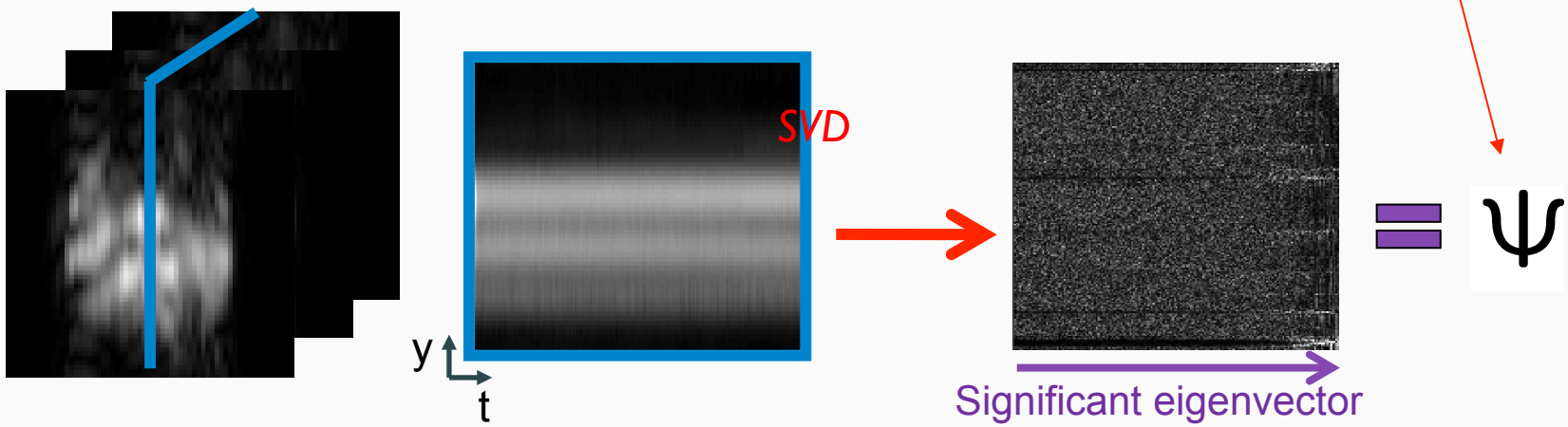
$$x_{n+1} = x_0 + \Theta_n A^H (A\Theta_n A^H + \lambda I)^{-1}(y - Ax_0)$$

Random phase encoding

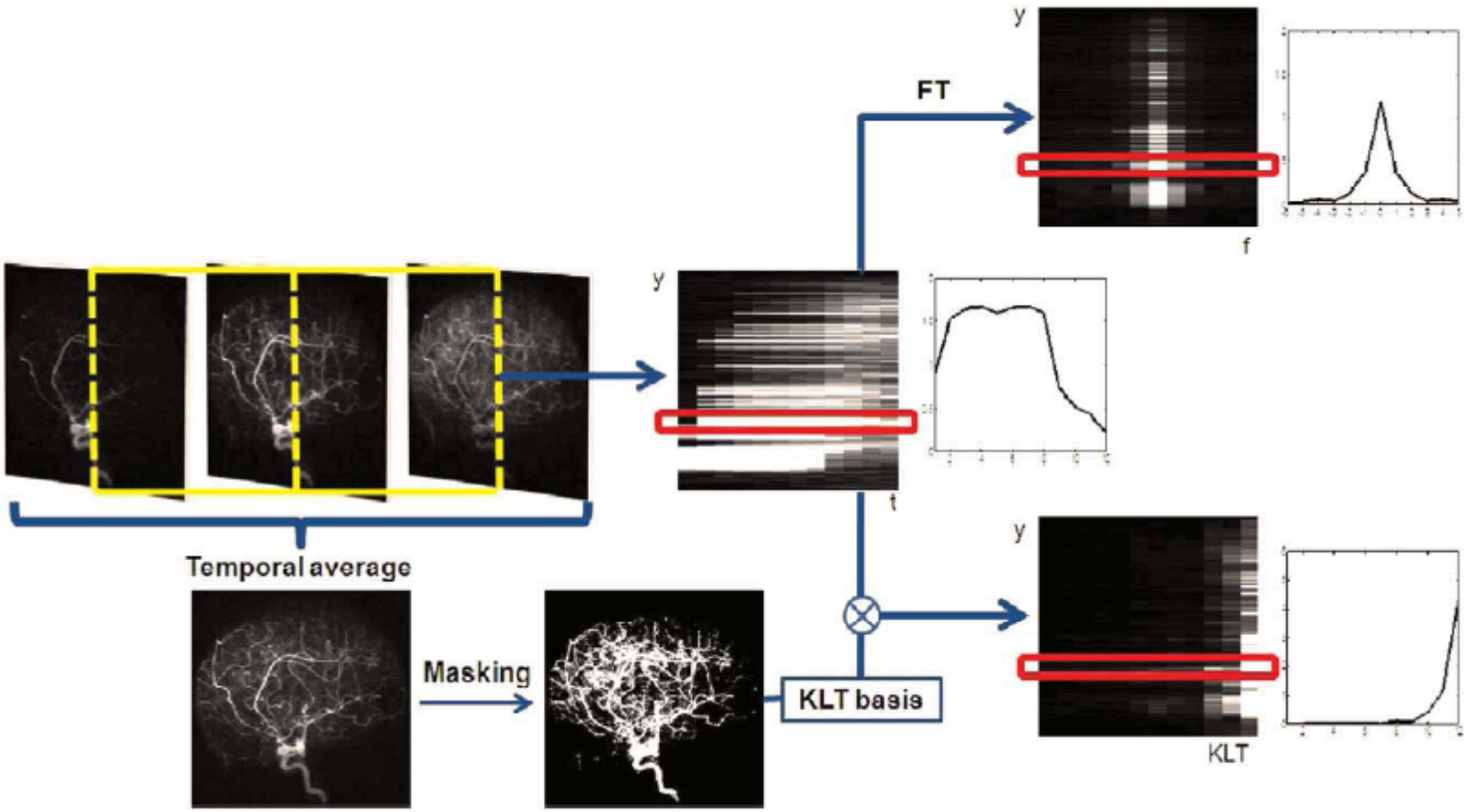
$$\mathbf{y} = \mathbf{A}\mathbf{x} = \Phi\Psi\mathbf{x}$$

k-t space sample

Temporal transform

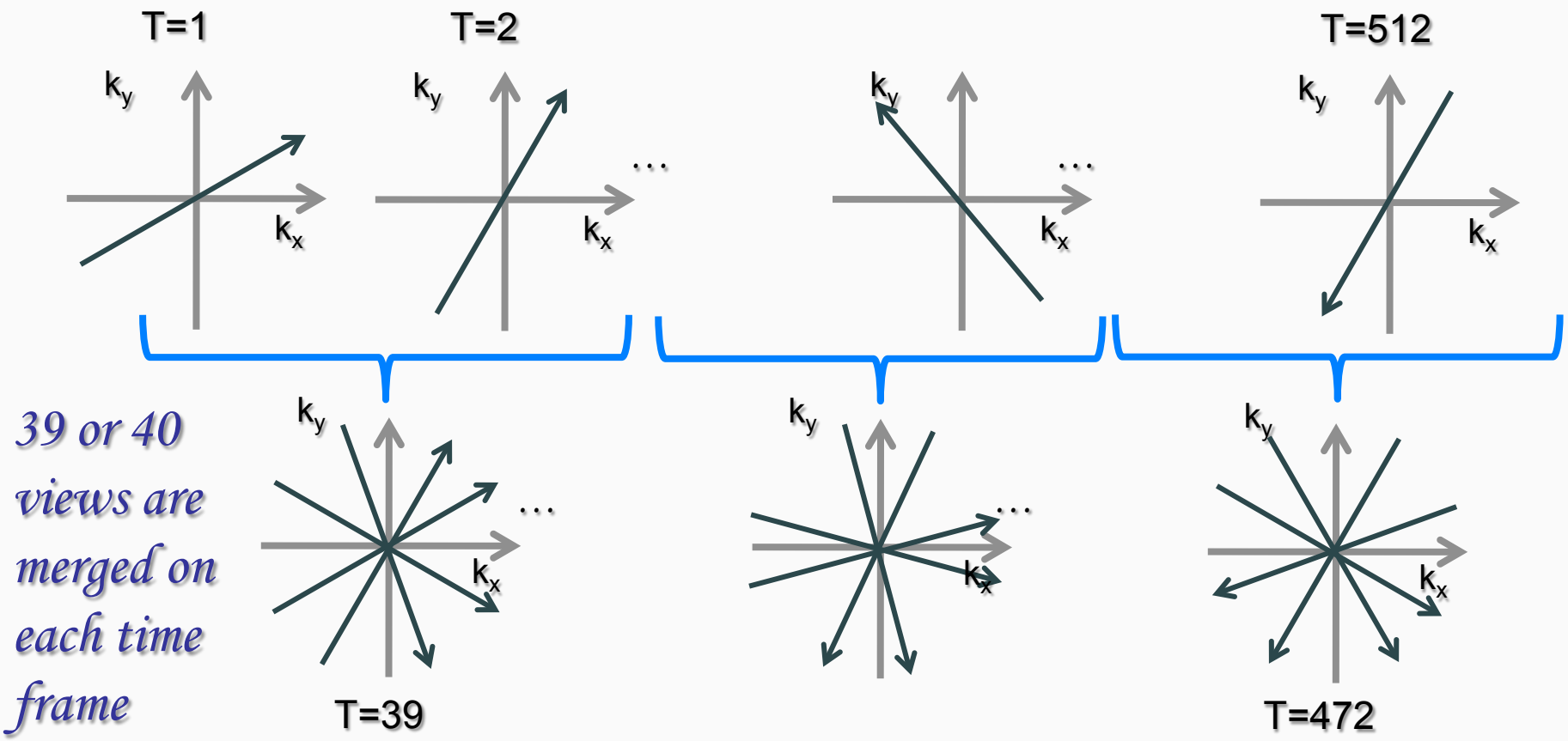


Two step Boosting for PCA basis



MR Angiography

Golden Section Radial Trajetory



39 or 40 views are merged on each time frame

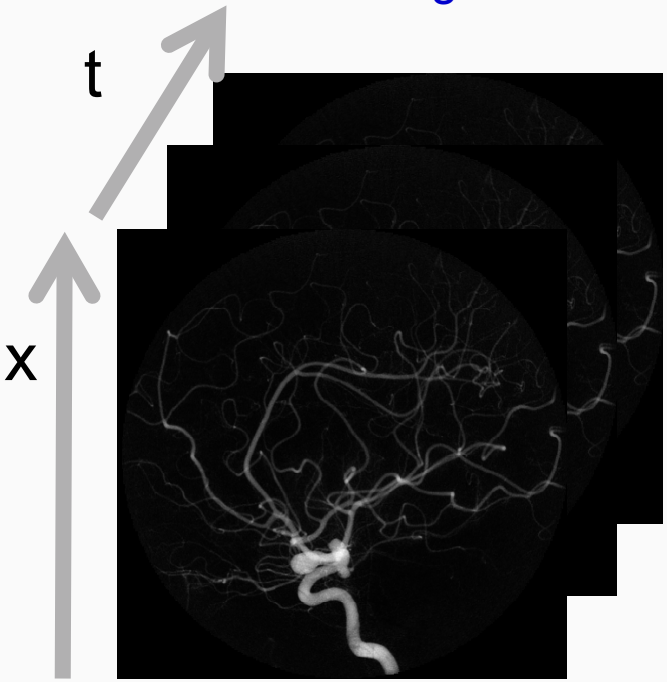
Target time points: *39, 79, 118, 157, 197, 236, 275, 314, 354, 393, 432, 472*

Measurement size (512 read-out x 512 views) vs Total recon matrix (512 x 512 x 12 time frames)

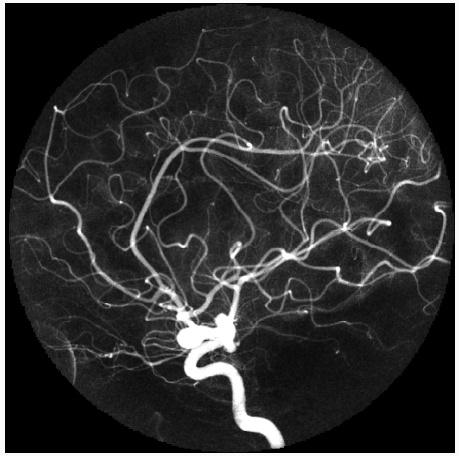
12 x acceleration !!

KLT for CE-MRA

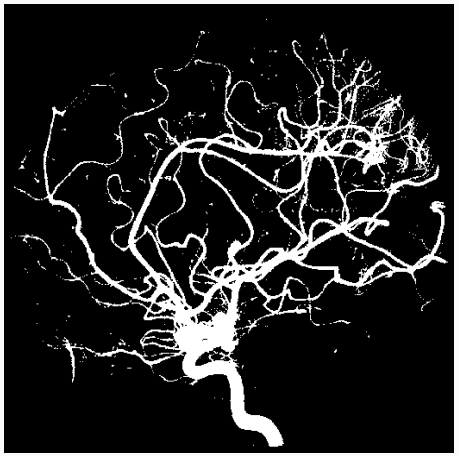
k-t FOCUSS recon using FFT

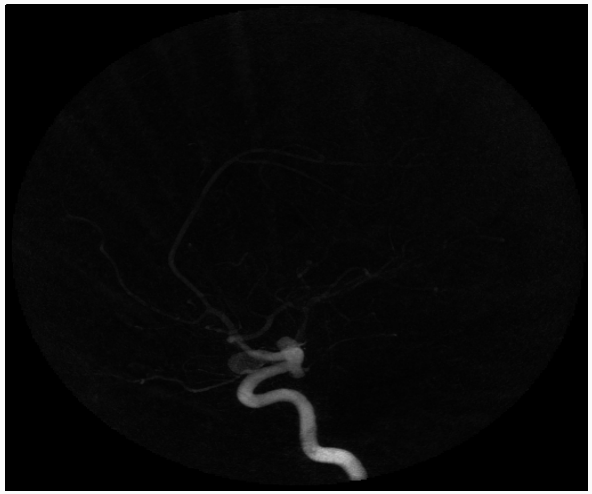


Summation along t-axis

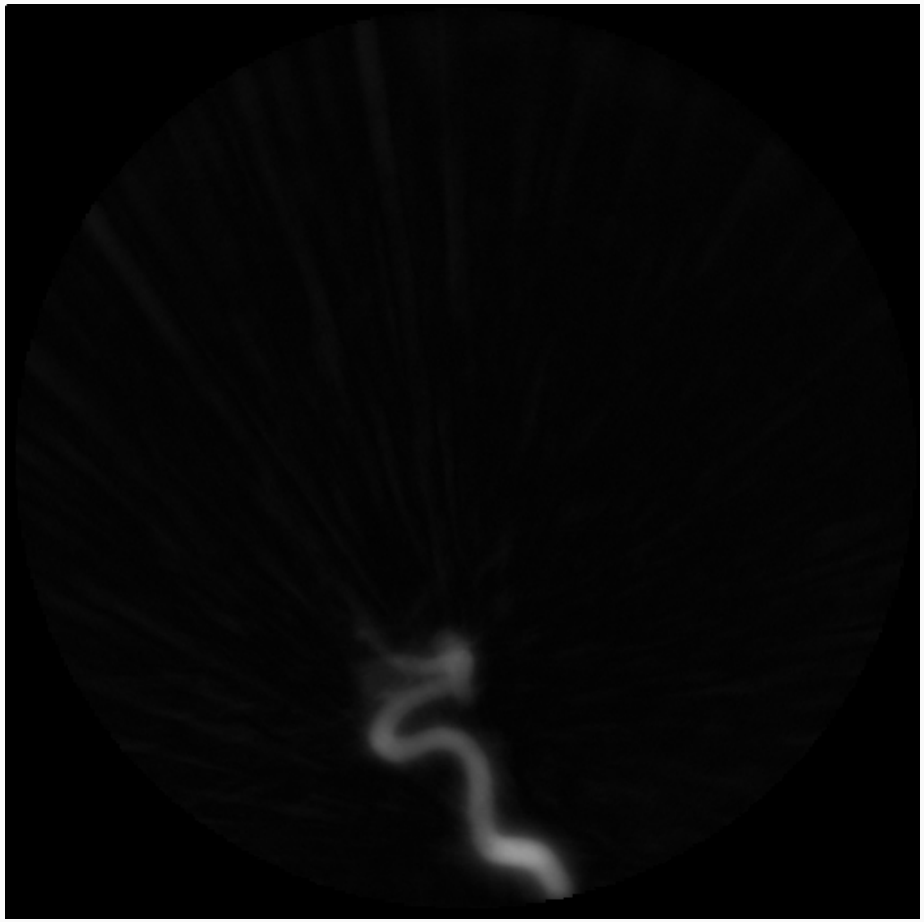


Thresholding

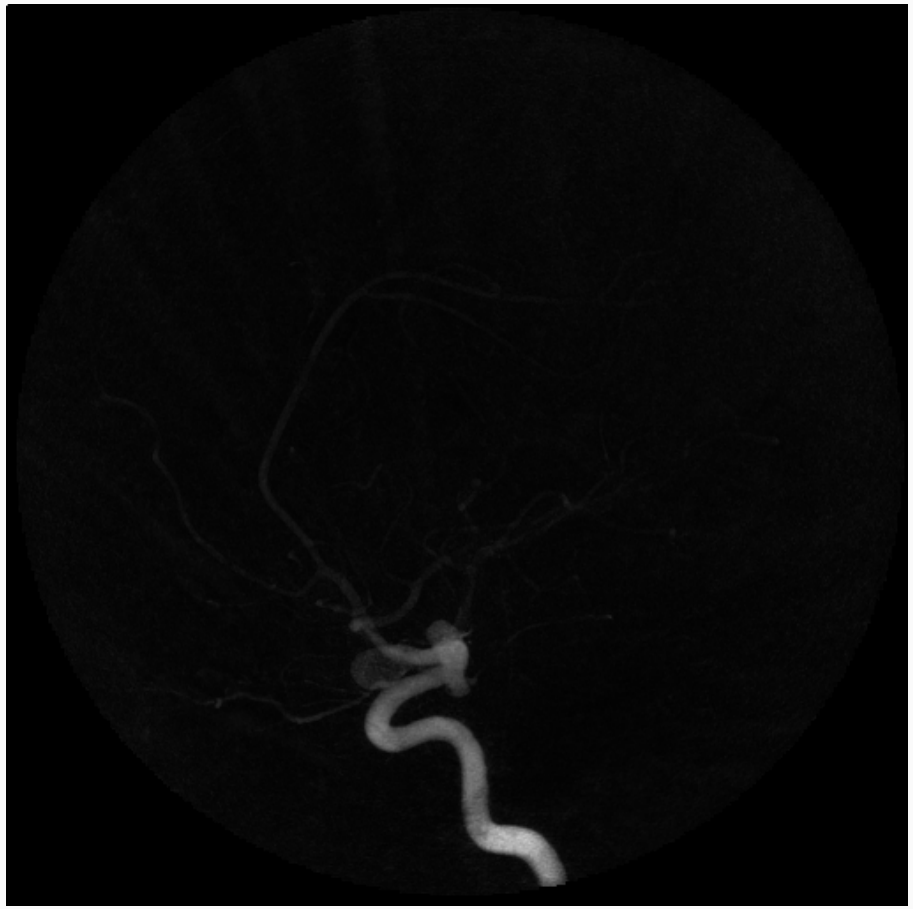




2009 ISMRM Recon Challenge Award



L2 minimization

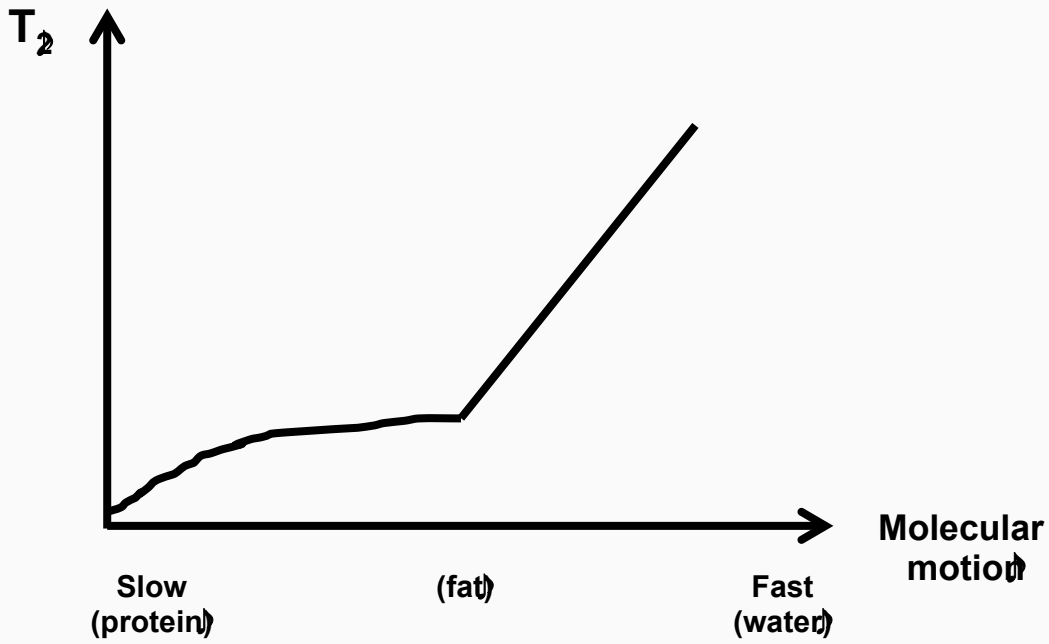


k-t FOCUSS using KLT

T2 Parameter Mapping

(joint work D. Kim, NYU: Feng et al, MRM, 2011)

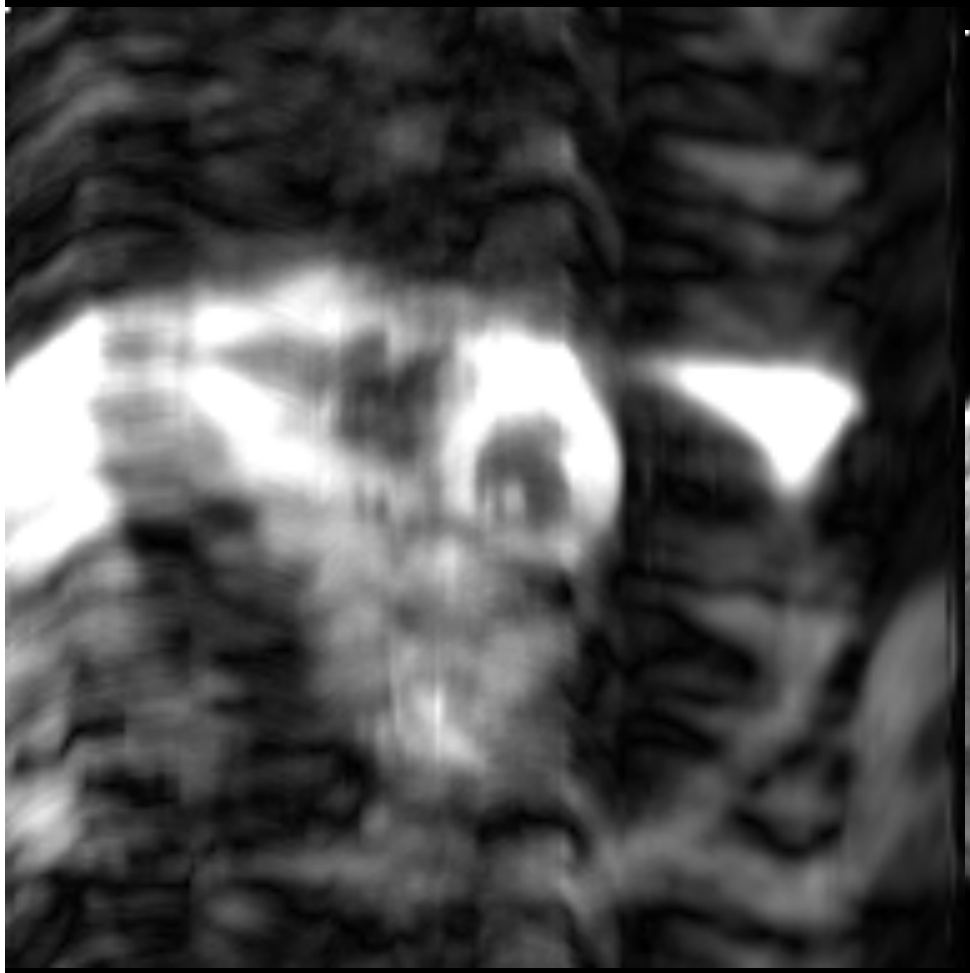
$$M_{xy}(t) = M_{xy}(0)e^{-t/T_2}$$



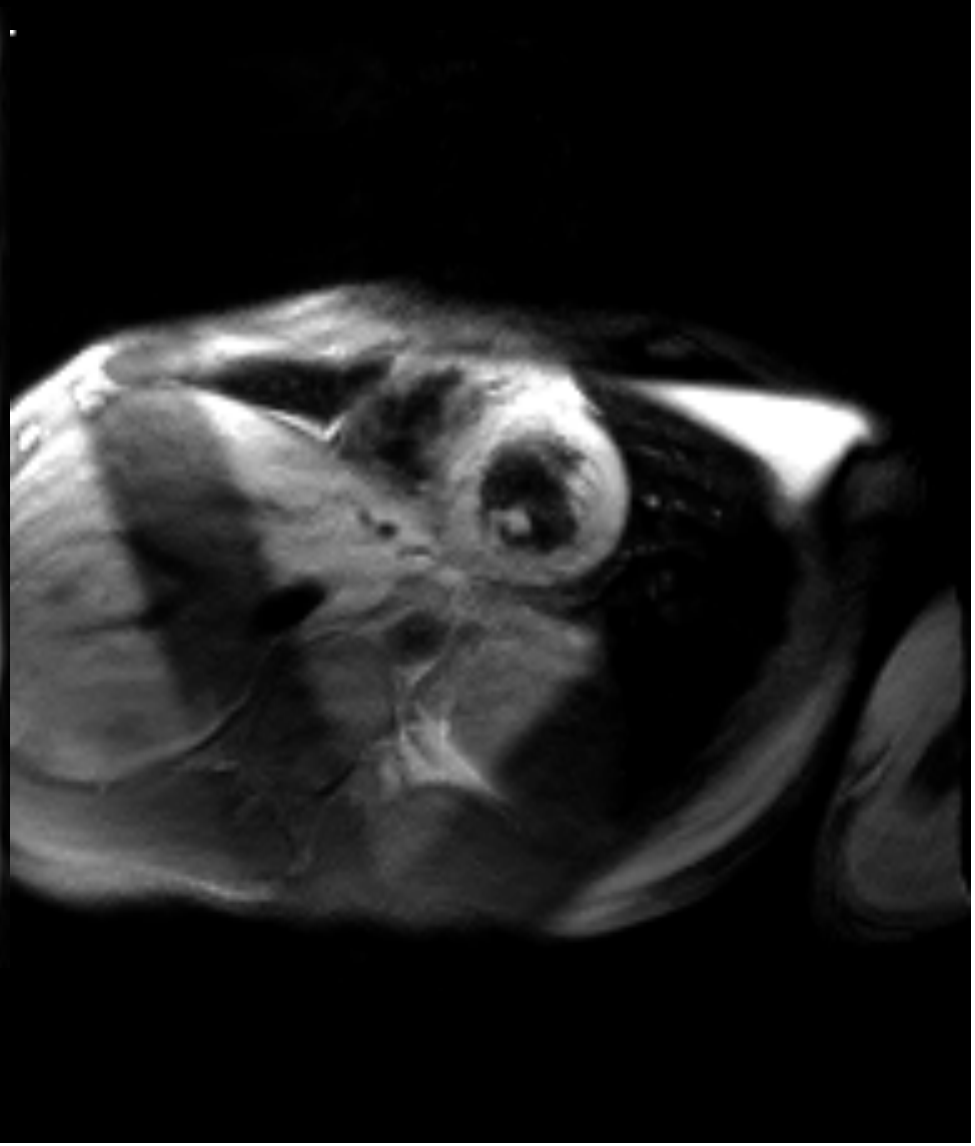
Detect T2 changes due to variety of disease

Cardiac T2 Mapping

6 x accel. Conventional method



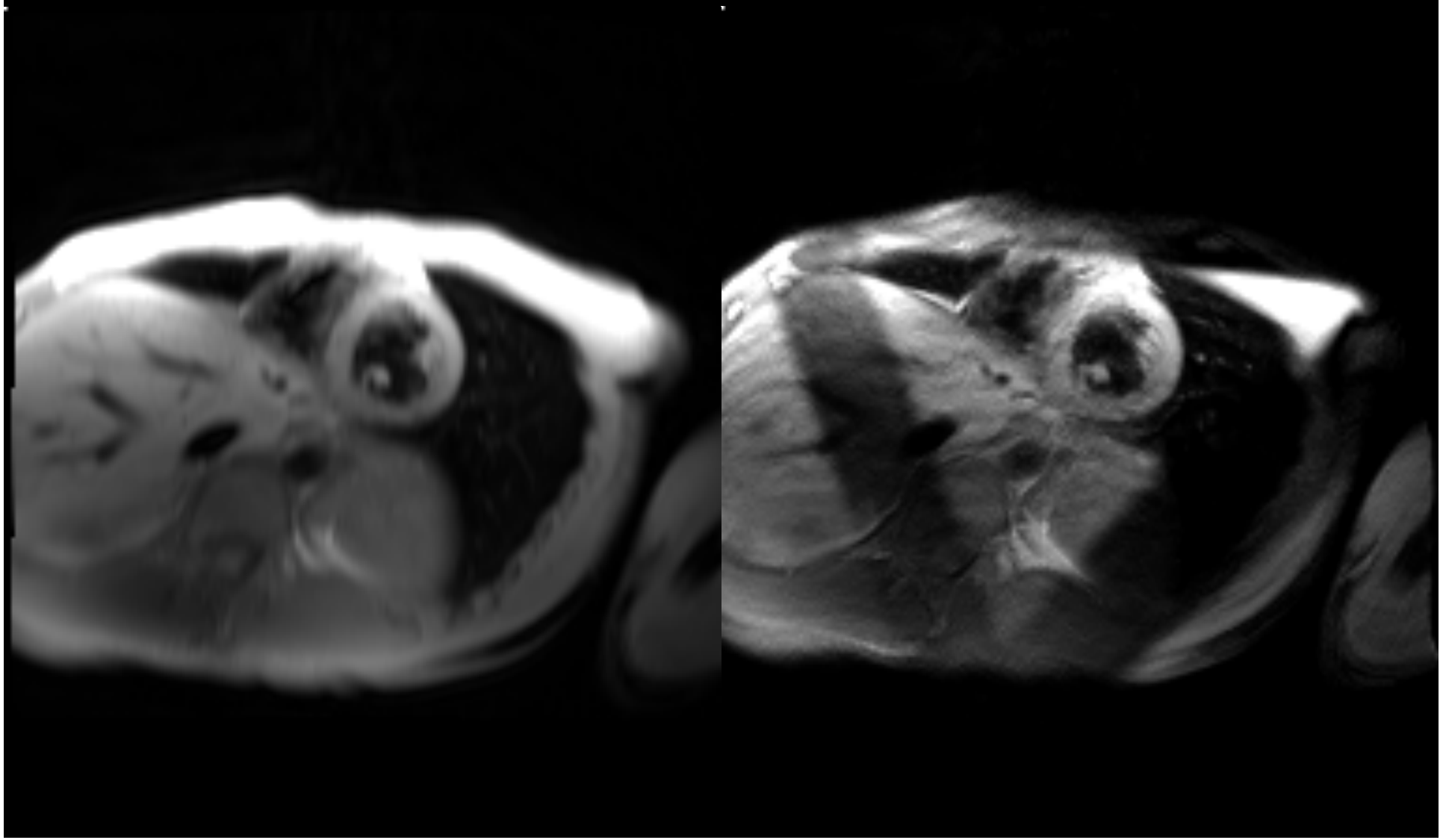
6 x accel. k-t FOCUSS



Cardiac T2 Mapping

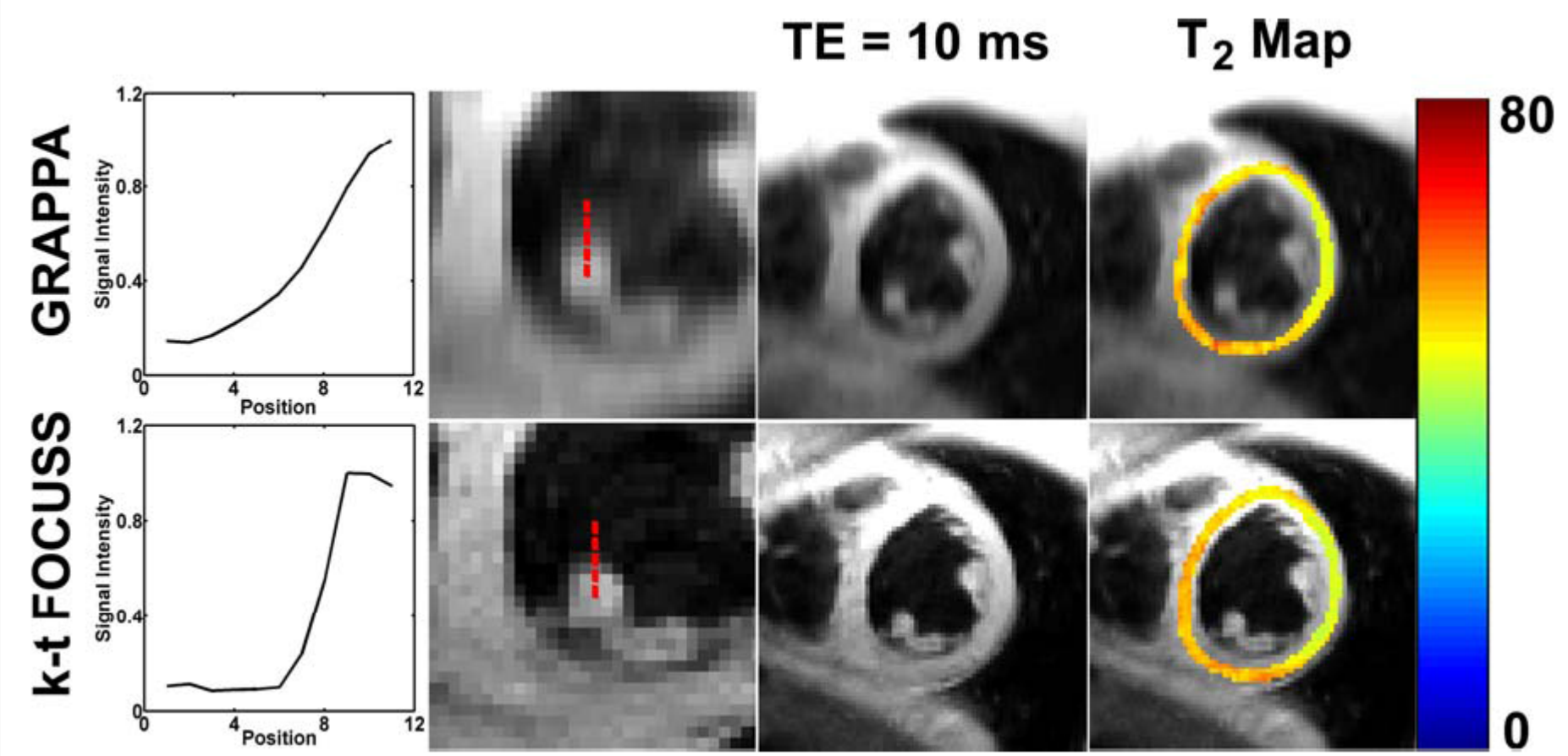
1.8 x accel. GRAPPA

6 x accel. k-t FOCUSS



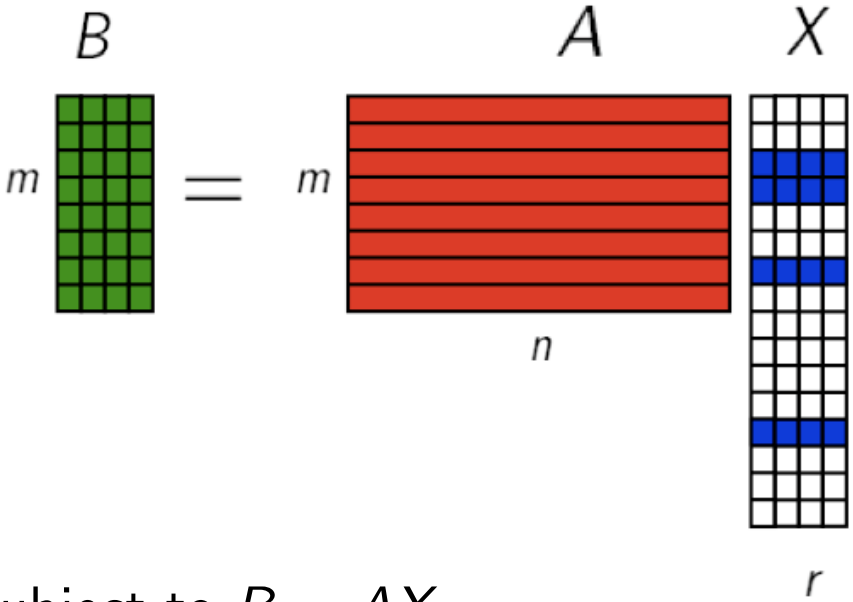
T2 Estimation

$$S(t) = (S_{ideal}^2 + \sigma^2)^{1/2} \qquad S_{ideal} \equiv S_0 e^{-t/T_2}$$



MULTIPLE MEASUREMENT VECTOR
COMPRESSED SENSING

MMV: Multiple Measurement Vector Problems



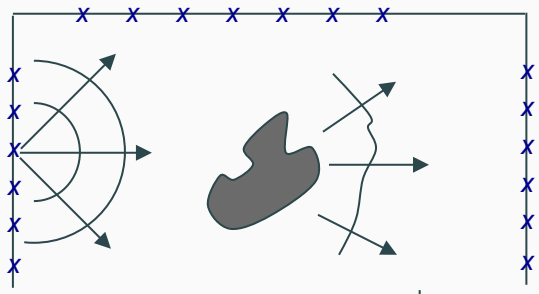
minimize $\|X\|_0$ subject to $B = AX$.

$k = \|X\|_0$: number of nonzero rows
r : number of snapshots
m : number of sensor elements
Here, we assume that the columns of B are linearly independent.

Inverse Scattering Problems

Geophysical

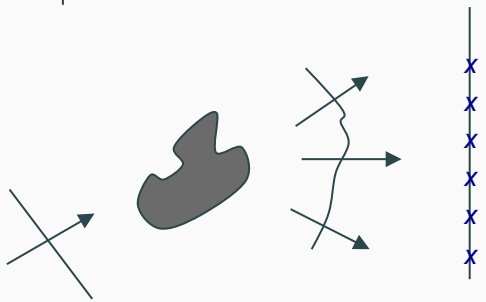
Electromagnetic
Acoustic



Off-set VSP/ cross-well tomography
GPR surface imaging
induction imaging

Medical

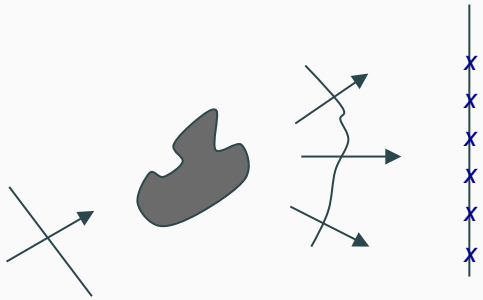
Ultrasonic
Optical



Ultrasound tomography
optical microscopy
photon imaging

Industrial

Electromagnetic
Ultrasonic
Optical

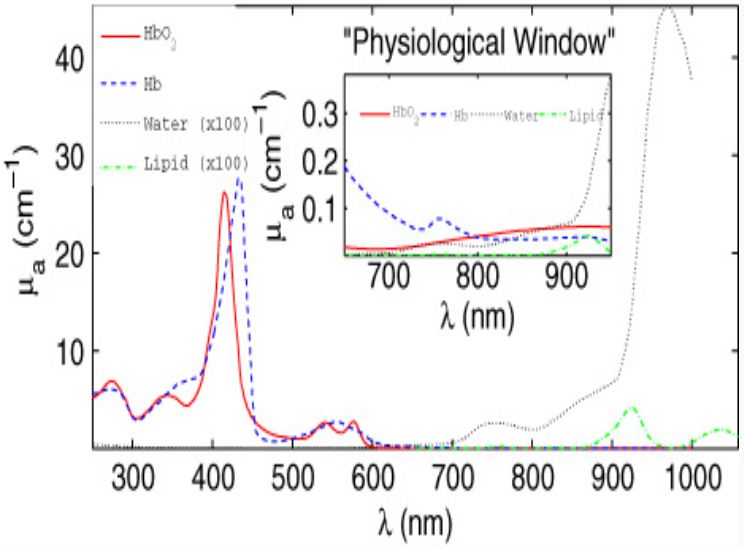


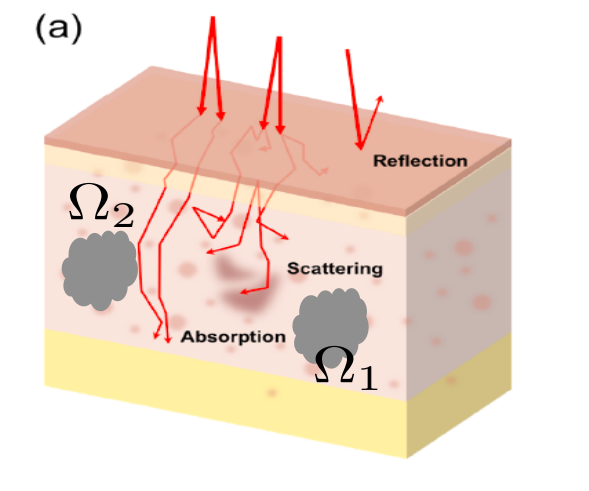
Ultrasound tomography
optical microscopy
induction imaging

Slide courtesy by Devaney

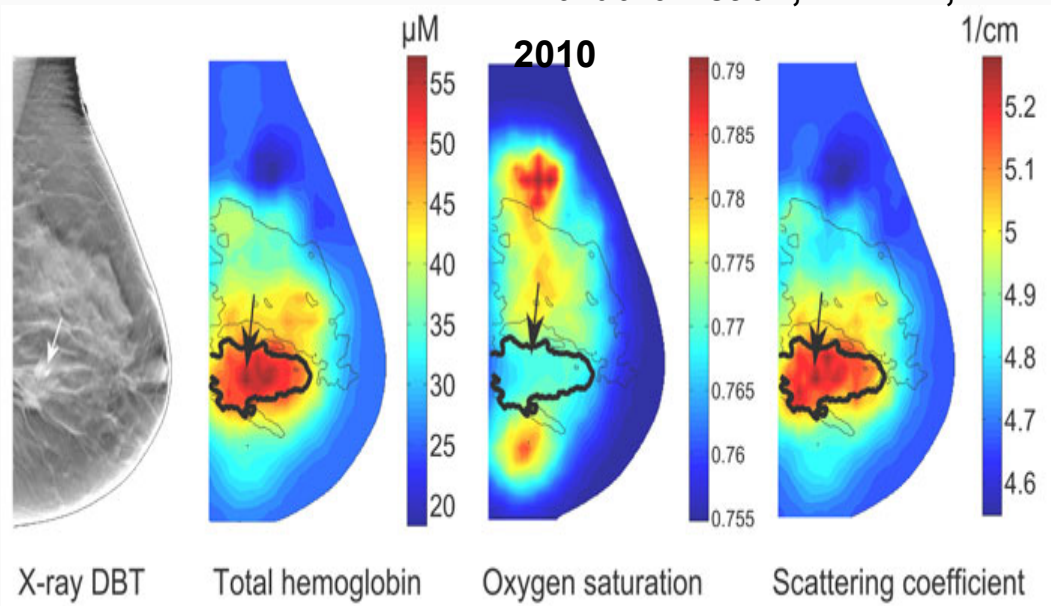
Diffuse Optical Tomography

Near-infrared (NIR, ~650-950 nm)



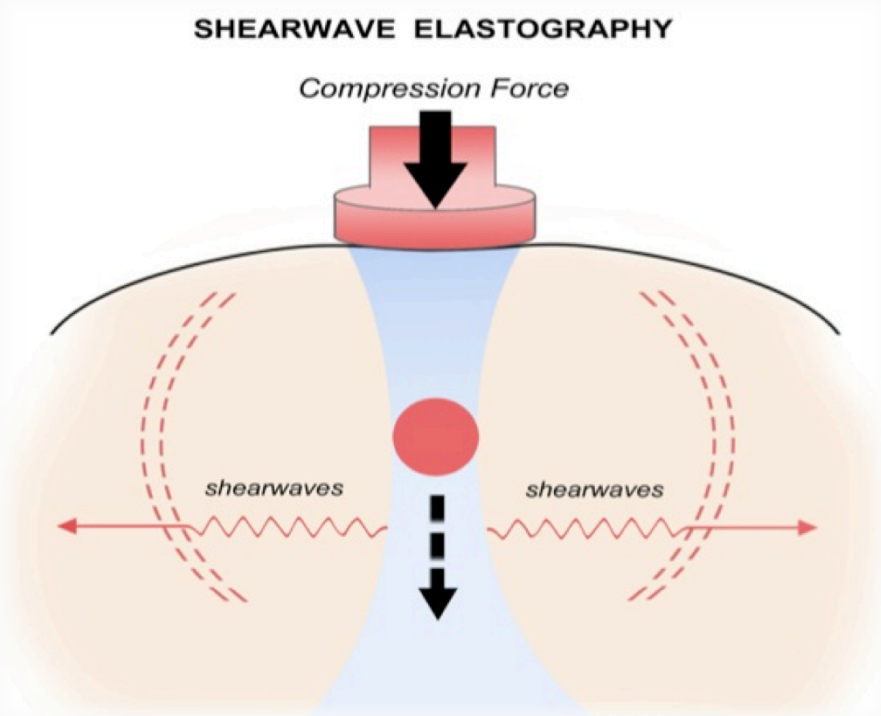


*** Durduran et al., MICCAI, 2010**

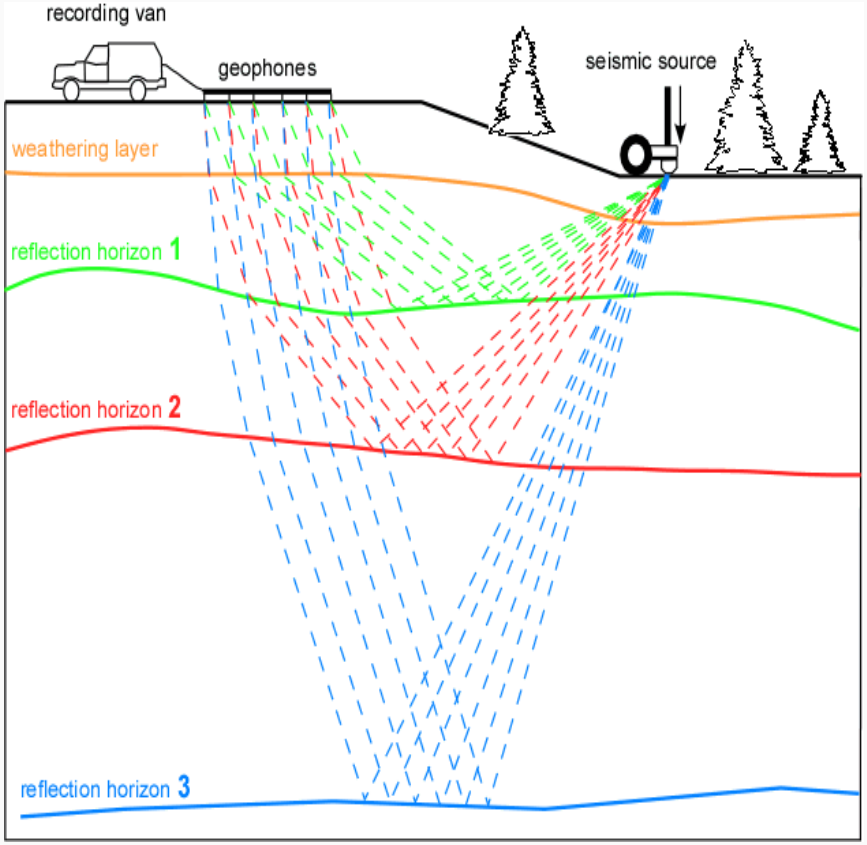


Elastic Source Imaging

Elastography: Medical, geophysical applications



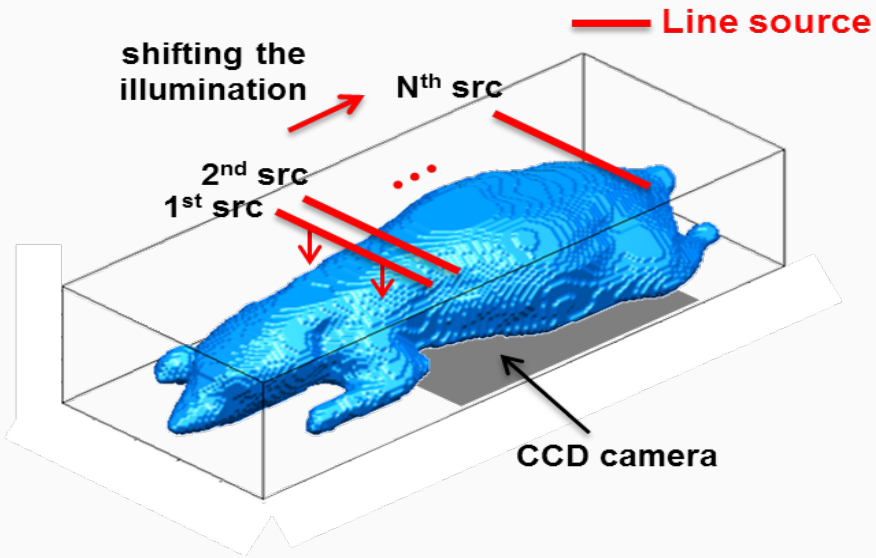
https://iame.com/online/breast_elastography/



https://marcellusdrilling.com/2018/04/9-more-seismic-testing-devices-stolen-in-swpa-6-were-returned/

Joint Sparsity

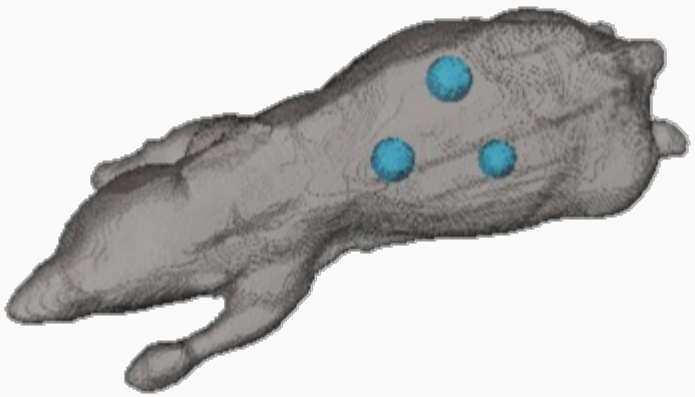
Multiple Measurement Vectors from Multiple illumination Patterns



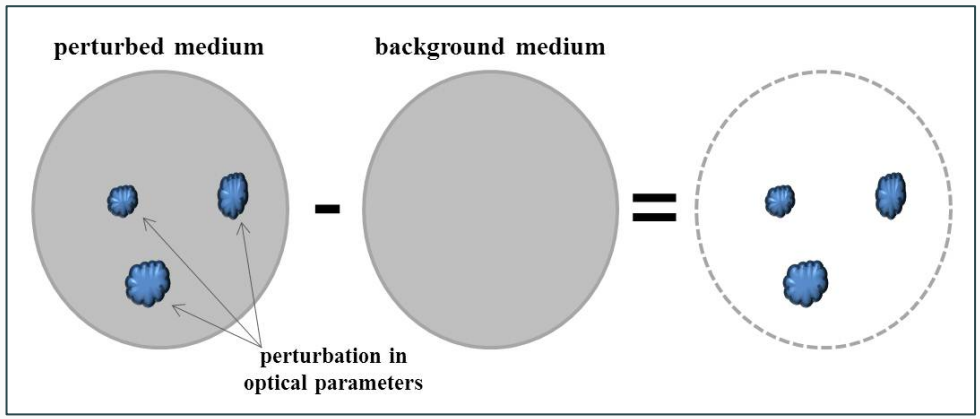
Finite # of snapshots are available

Joint Sparsity

Sparse Signal ➡ Perturbations in the optical parameters

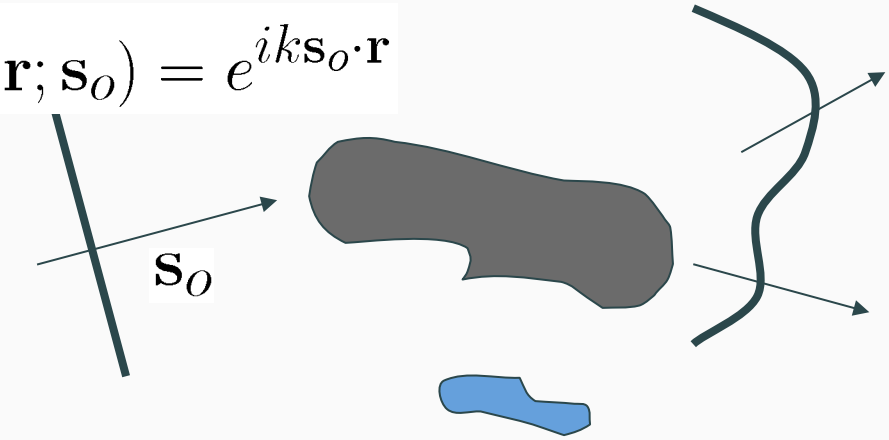


Angiogenesis in cancer



Inverse Scattering Problems

$$\psi^{(in)}(\mathbf{r};\mathbf{s}_o) = e^{ik\mathbf{s}_o\cdot\mathbf{r}}$$



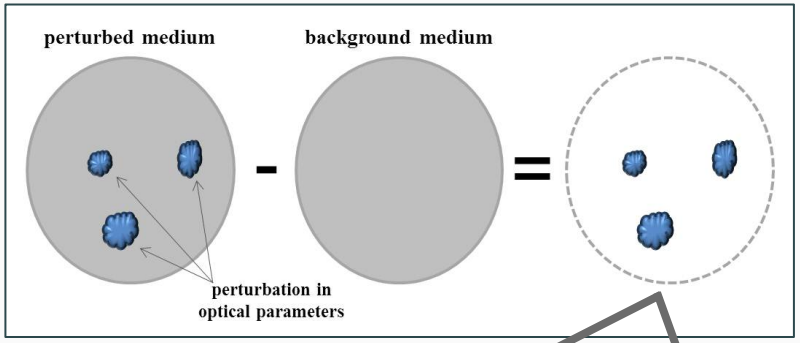
$$\psi = \psi^{(in)} + \psi^{(s)}$$

$$u_m^s(\mathbf{x}) = -\frac{1}{D_0}\int_{\cup_{n=1}^N \Omega_n} \overbrace{G^{\kappa_0}(\mathbf{x},\mathbf{y})}^{\text{Ill-posed}} \underbrace{u_m^t(\mathbf{y})f(\mathbf{y})}_{\text{Non-linear}} d\mathbf{y},$$

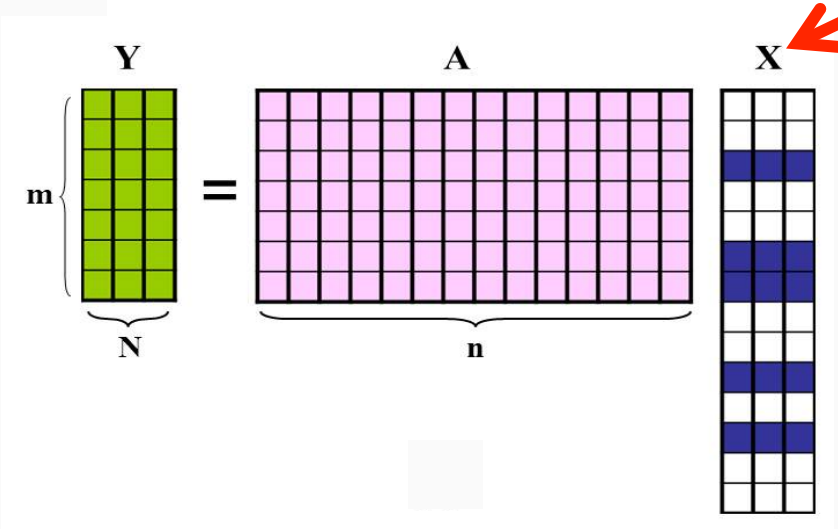
Existing methods

- Born approximation → linearization error
- Iterative Born approximation → computationally expensive

Overcoming Nonlinearities using Joint Sparsity



$$u_m^s(\mathbf{x}) = -\frac{1}{D_0}\int_{\cup_{n=1}^{N}\Omega_n} G^{\kappa_0}(\mathbf{x},\mathbf{y})\boxed{u_m^t(\mathbf{y})f(\mathbf{y})}d\mathbf{y},$$



L_0 Uniqueness of MMV

Definition

Given a matrix A, let spark(A) denote the smallest number of linearly dependent columns of A.

Theorem (Rao / Chen, Huo / Feng, Bresler / Davies, Eldar)

If a matrix X satisfies $AX = B$, then

$$\|X\|_0 < \frac{\mathrm{spark}(A) + \mathrm{rank}(B) - 1}{2}$$

if and only if X is the unique solution to the l_0 minimization problem.

With increasing number of snapshots, more non-zero elements can be recovered.

Spark Reduction Principle

1. J. Kim, O. Lee, J. Ye, Compressive MUSIC (CS-MUSIC), *IEEE Trans. on Inform. Theory*, January 2012

Theorem

Let $r \leq m < n$. Suppose that we are given a sensing matrix $A \in \mathbb{R}^{m\times n}$ and an observation matrix $B \in \mathbb{R}^{m\times r}$. If the k nonzero rows of X are in general position (i.e., any collection of r nonzero rows are linearly independent) and A satisfies the RIP condition $0 \leq \delta^L_{2k-r+1}(A) < 1$, then

$$\mathrm{spark}(Q^*A) = k - r + 1.$$

Note that $A \in \mathbb{R}^{m\times n}$ satisfies RIP with $0 \leq \delta^L_{2k-r+1}(A) < 1$ if and only if

$$k < \frac{\mathrm{spark}(A) + \mathrm{rank}(B) - 1}{2}.$$

:l_0 bound for MMV problem

CS-MUSIC achieves l_0 bound with finite snapshot.

Generalized MUSIC Criterion

Theorem

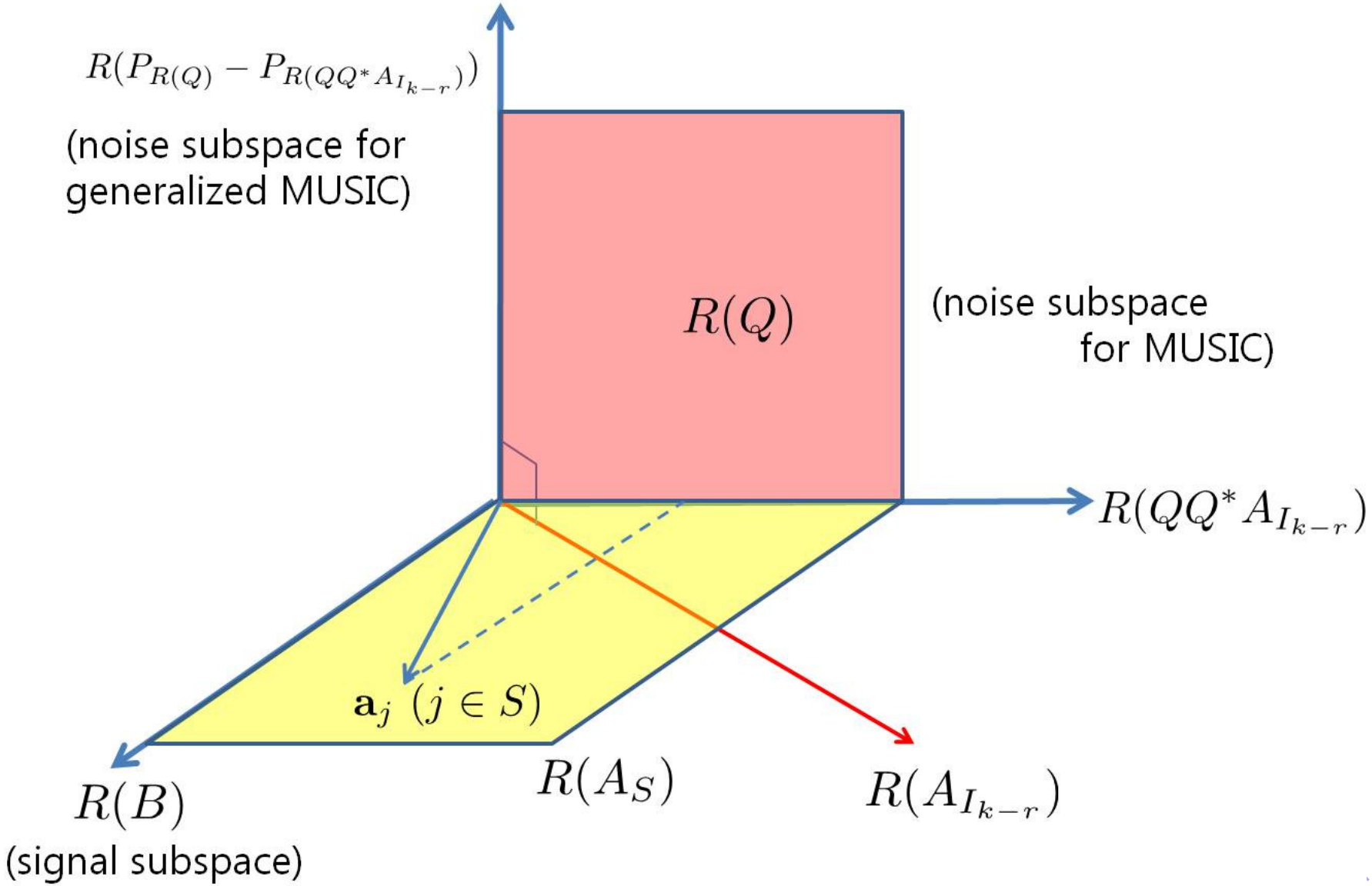
Assume that we have an MMV problem $AX = B$, where A, X and B as in the previous theorem. If $I_{k-r} \subset \mathrm{supp}X$ with $|I_{k-r}| = k - r$ and $A_{I_{k-r}} \in \mathbb{R}^{m\times(k-r)}$ which consists of columns of A, whose indices are in I_{k-r}, then for any $j \in \{1, \cdots, n\} \setminus I_{k-r}$,

$$\mathbf{a}_j^* \left[P_{R(Q)} - P_{R(P_{R(Q)} A_{I_{k-r}})} \right] \mathbf{a}_j = 0 \Longleftrightarrow j \in \mathrm{supp}X.$$

$$\mathbf{a}_j^* P^{\perp}_{[A_{I_{k-r}} B]} \mathbf{a}_j = 0 \Longleftrightarrow j \in \mathrm{supp}X.$$

Augmented Signal Subspace: $R[A_{I_{k-r}} B]$

Geometric Interpretation



Sampling Rate for MMV

Case 1: r is a fixed number

Theorem

Let $\frac{\sigma_k\left([A_{I_{k-r}} \;\; S]\right)}{\Delta} > 1 + \frac{k}{r}$. *If we have*

$$m > \frac{1+\delta}{1 - 2\frac{k}{r}\eta_{k-r}} \frac{2k\log(n-k)}{r},$$

then we can find $k - r$ *correct indices of* $\mathrm{supp}X$ *by applying subspace S-OMP.*

1. [Fletcher, Rangan] For SMV, when SNR $\to \infty$, we need $m > 2(1+\delta)k\log(n-k)$ for some $\delta > 0$.
2. The sampling ratio is reciprocally proportional with respect to the number of multiple measurement vectors.
3. SNR$\to \infty$ is required, in the large system limit. (Similar to Reevs and Gaspar)

Sampling Rate for MMV

Case 2: $\alpha := \lim_{n\to\infty} r(n)/k(n) > 0$

Theorem

Let $\frac{\sigma_k\left([A_{I_{k-r}} \;\; S]\right)}{\Delta} > 1+\alpha$ *Then if we have*

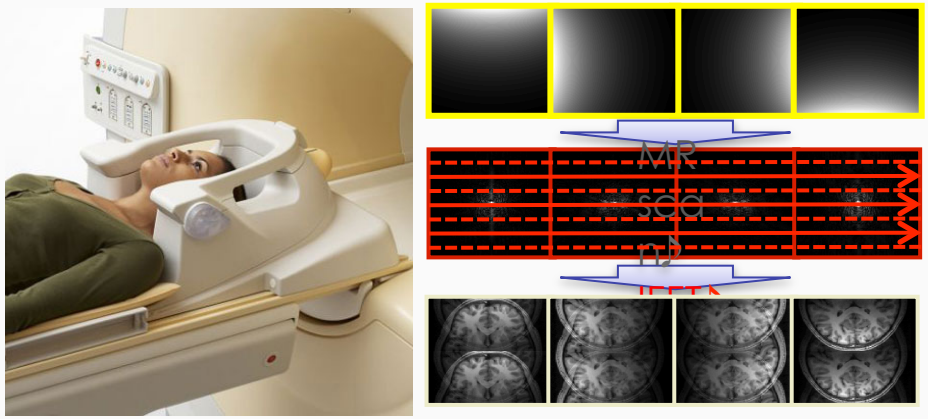
$$m > (1+\delta)^2 \frac{k}{1-2\frac{\eta_{k-r}}{\alpha}} \left[2 - F(\alpha)\right]^2,$$

for some $\delta > 0$ *where* $F(\alpha)$ *is an increasing function on* $[0,1]$ *such that* $F(1) = 1$ *and* $\lim_{\alpha\to 0^+} F(\alpha) = 0$. *Then we can find* $k-r$ *correct indices of* $\mathrm{supp}X$ *by applying subspace S-OMP.*

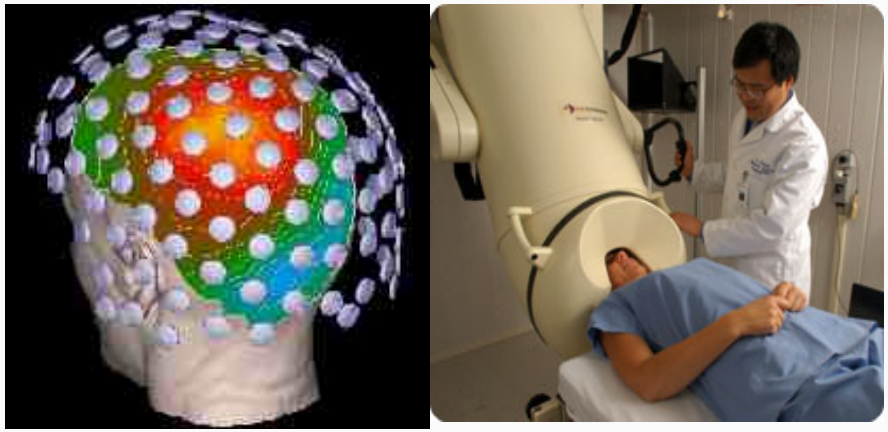
1. If $\alpha \to 1$ and SNR $\to \infty$, then we only need to have $m > (1+\delta)k$ for a small $\delta > 0$. (cf. MUSIC)
2. In this case, the required number of sensor elements is at most $4(1+\delta)k$ when SNR$\to \infty$ (No log n factor).
3. The required SNR is finite.

MMV for Bio-medical Imaging

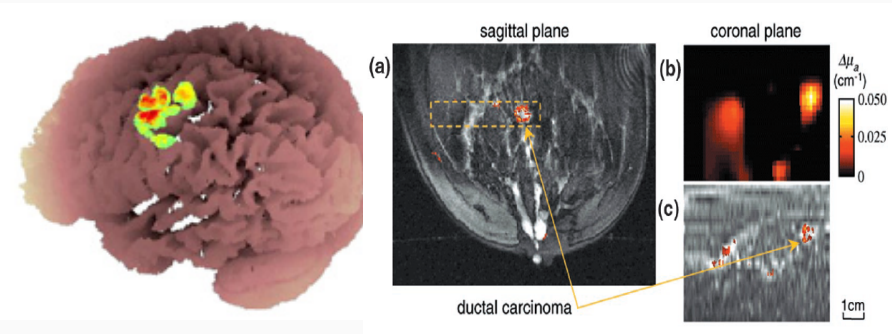
- Parallel MRI + CS



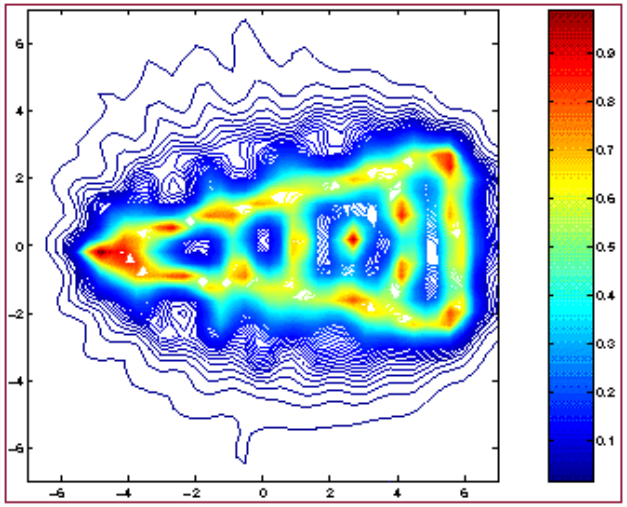
- EEG/MEG



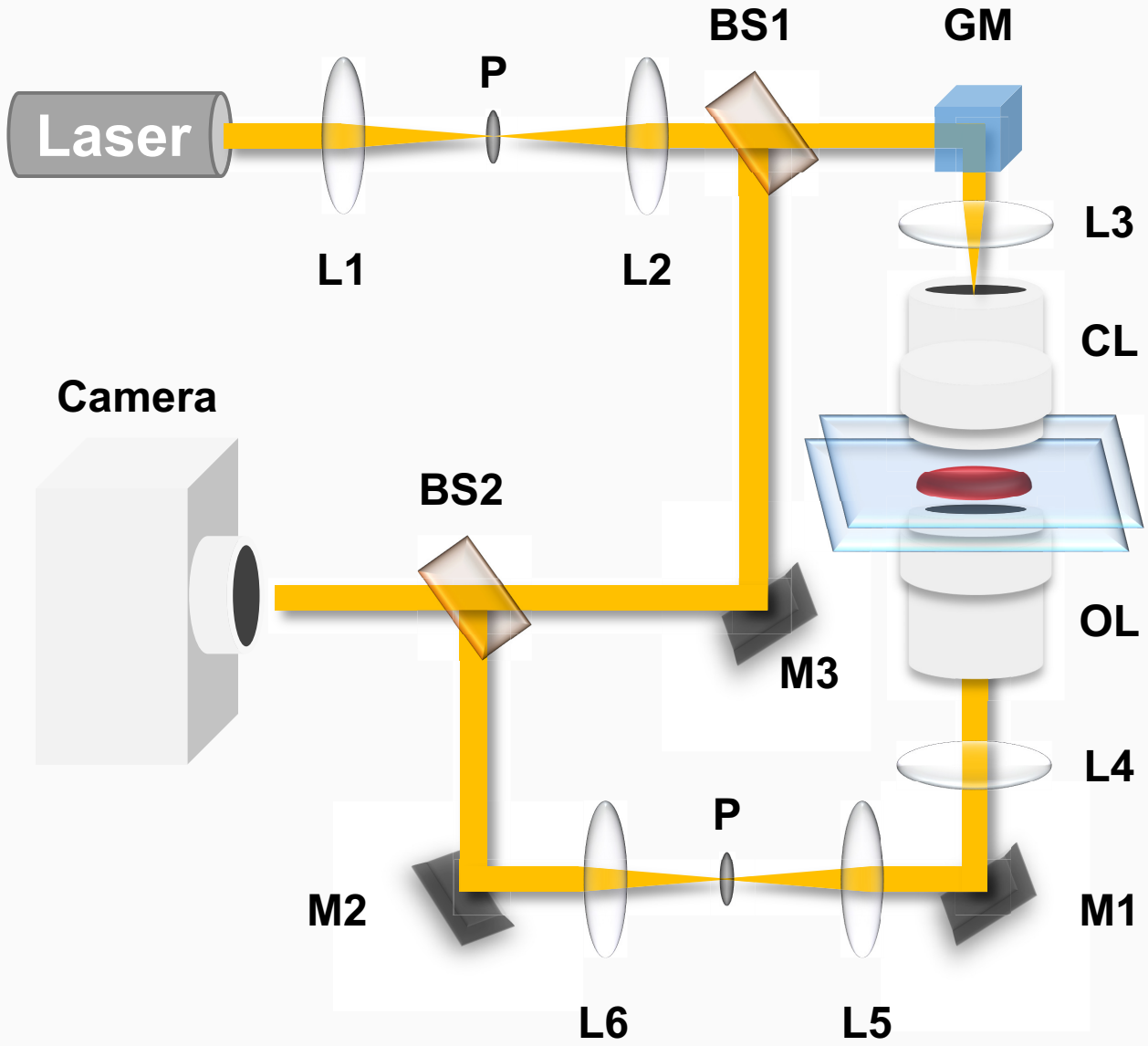
- Diffuse optical tomography



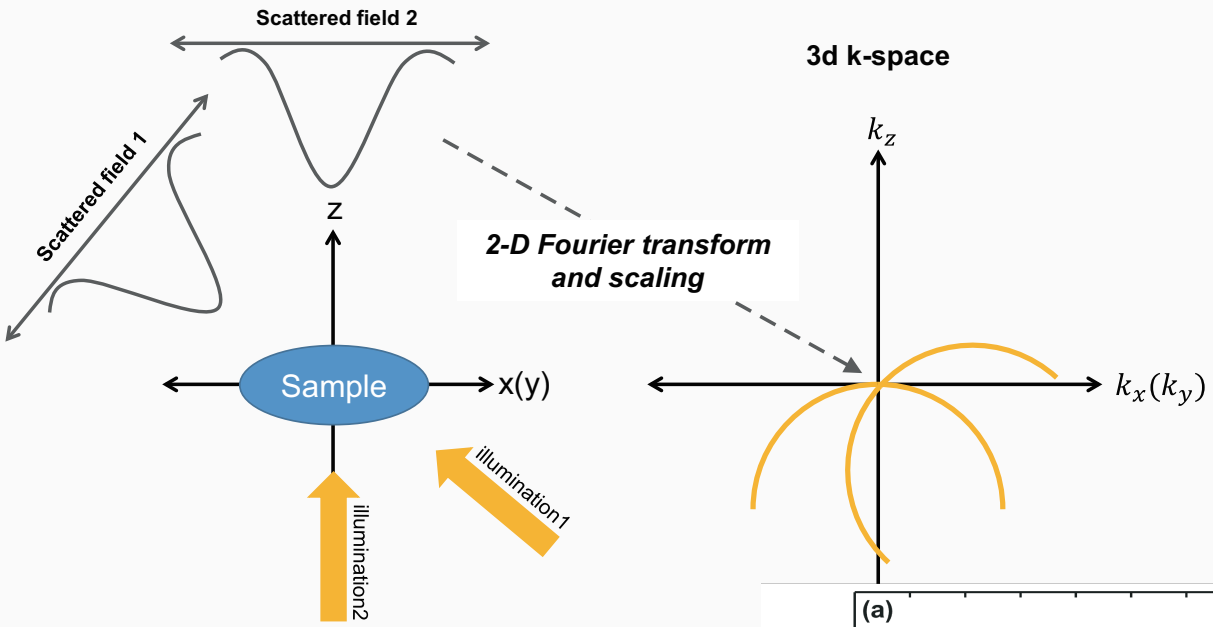
- Wave inverse scattering

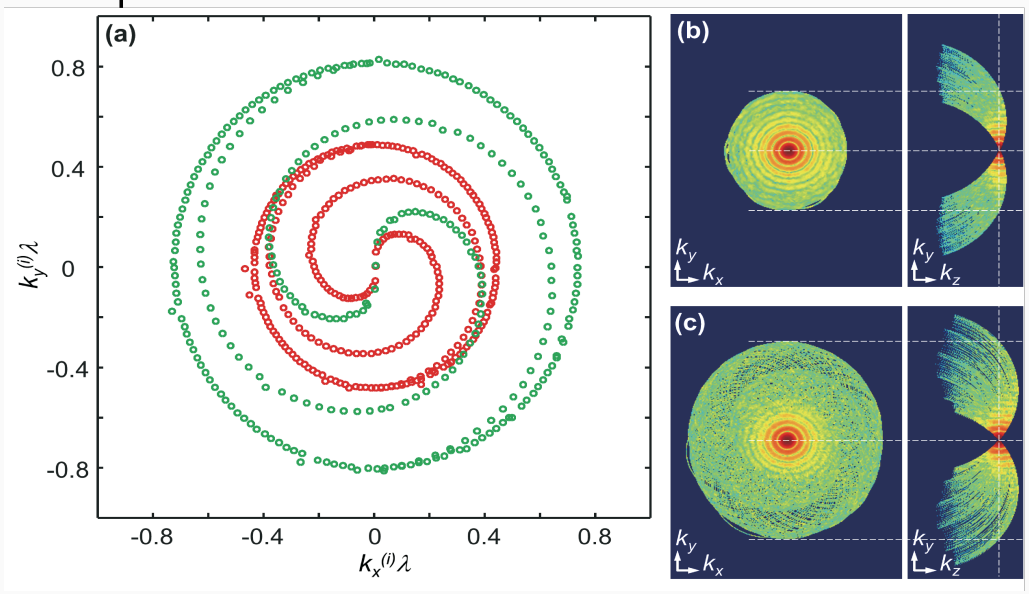


Optical Diffraction Tomography (ODT)



ODT under Born/Rytov Approximation





ODT using Joint Sparsity

Lippman-Schwinger equation

$$U_s(\mathbf{r}) = \tau K_D[U](\mathbf{r}) := \tau \int_D \kappa^2 f(\mathbf{r}')G(\mathbf{r},\mathbf{r}')U(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in \overline{\Omega},$$

MMV for Support & Induced Current Recovery

$$\begin{cases} U_s^{(m)}(\mathbf{r}) = \tau\kappa^2 \int_D G(\mathbf{r},\mathbf{r}')I^{(m)}(\mathbf{r}')d\mathbf{r}', & \mathbf{r} \in \Gamma, \\ I^{(m)}(\mathbf{r}) = f(\mathbf{r})U^{(m)}(\mathbf{r}), & m = 1,\cdots,M, \end{cases}$$

$$\min_{\mathcal{I}} \|\mathcal{I}\|_0, \text{ subject to } \|\mathcal{Y} - \mathcal{G}\cdot\mathcal{I}\|_F \le \epsilon.$$

ODT using Joint Sparsity

Unknown Total Field Estimation

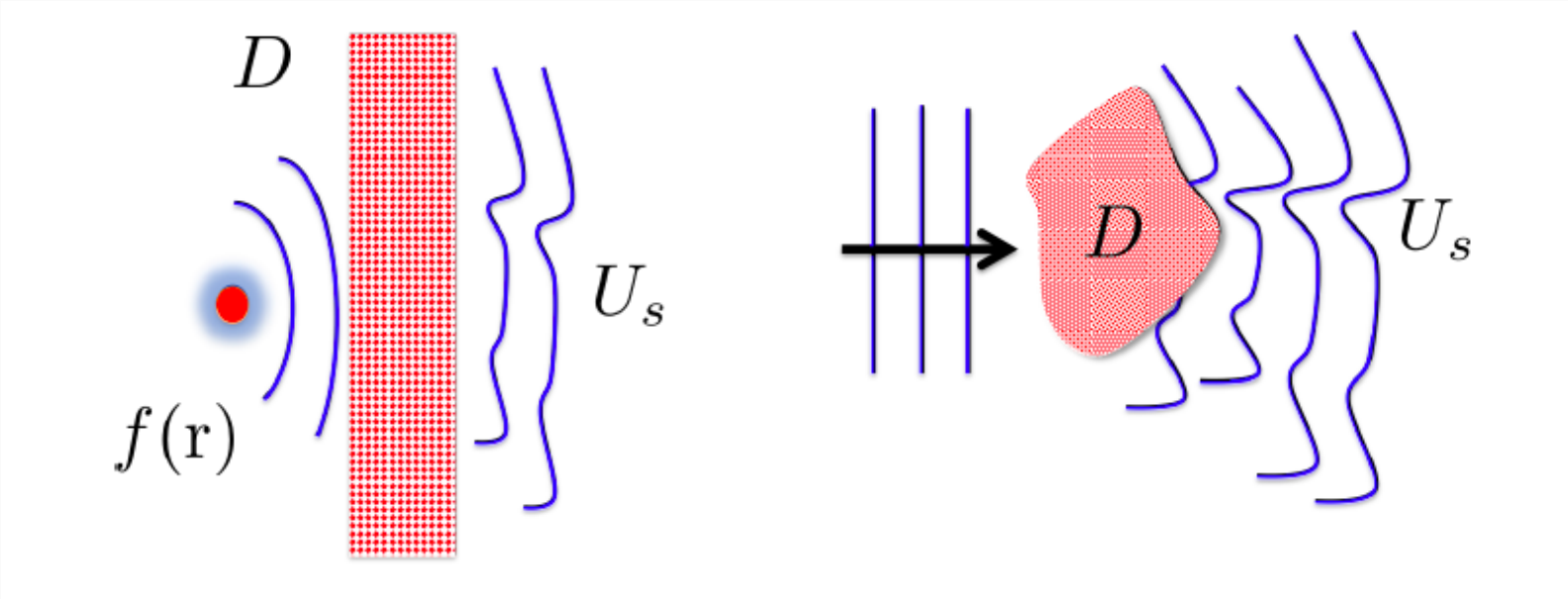
$$\hat{U}(\mathbf{r}) = U_0(\mathbf{r}) + \tau\kappa^2 \int_{\hat{D}} G(\mathbf{r},\mathbf{r}')\hat{I}(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in D,$$

Abnormality Estimation

$$U_s(\mathbf{r}) \quad = \quad \tau\kappa^2 \int_{\hat{D}} G(\mathbf{r},\mathbf{r}')\hat{U}(\mathbf{r}')f(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in \Gamma.$$

Super-Resolution from Joint Sparse Recovery

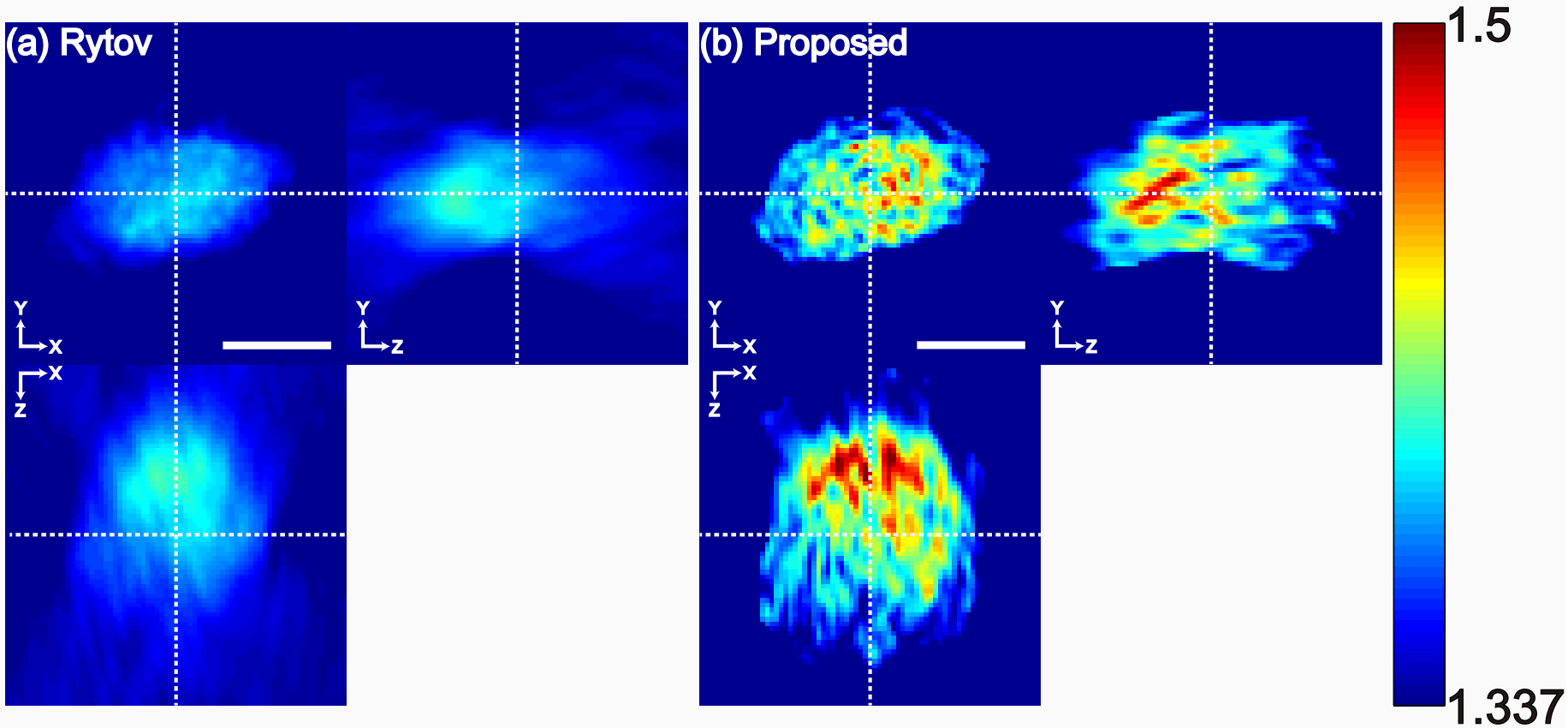
(Lim et al, OPEX, 2017)



$$u_s(\mathbf{r}) = \sum_i a_i (1/\tau' - K_{D'})^{-1} K_{D'}\left[\varphi_i\right](\mathbf{r}) = \sum_i a_i \frac{\lambda_i}{1/\tau' - \lambda_i} \varphi_i(\mathbf{r}), \quad \mathbf{r} \in D',$$

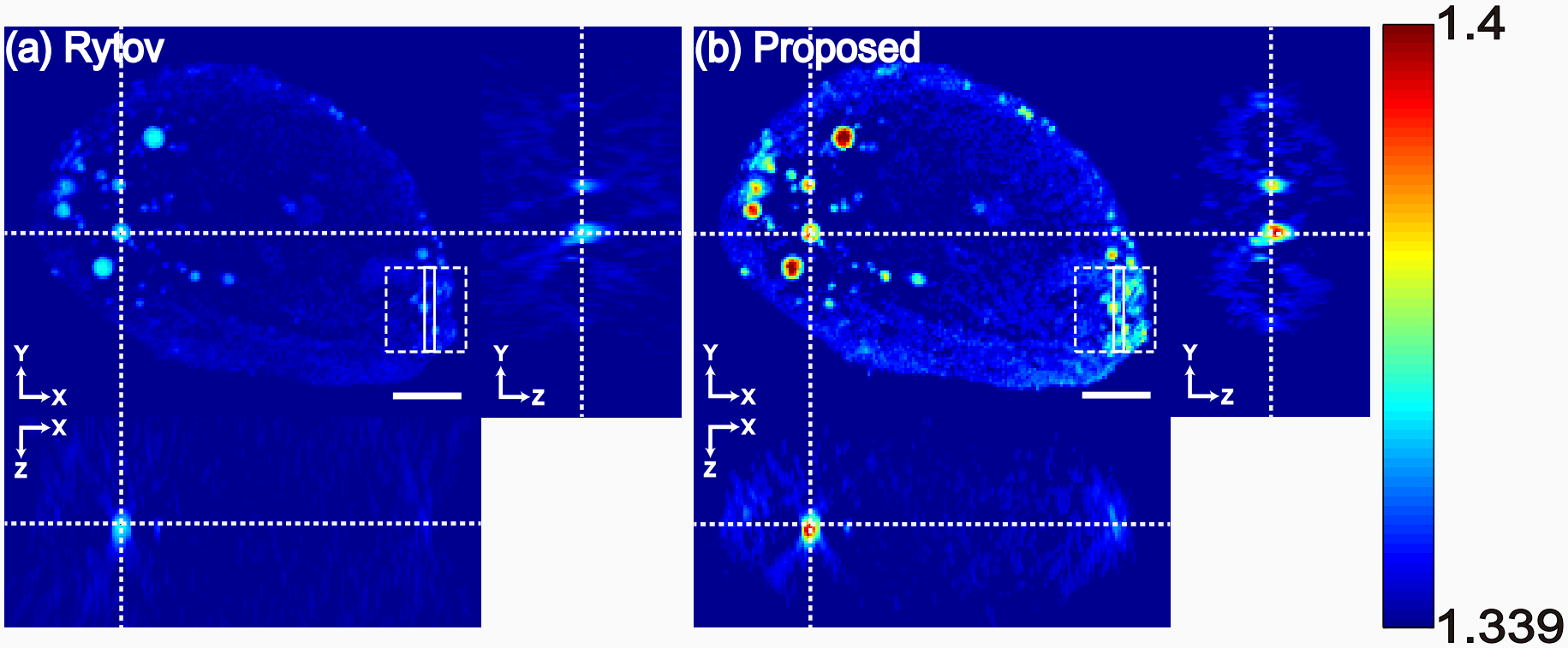
Super-Resolution from Joint Sparse Recovery

(Lim et al, OPEX, 2017)



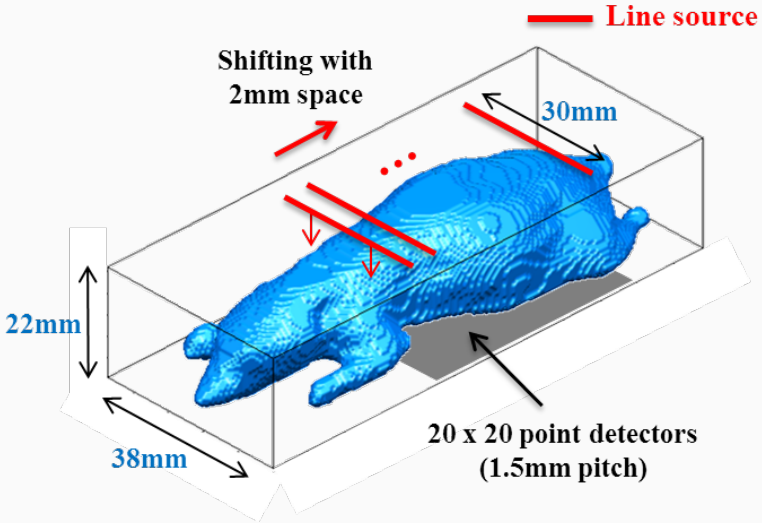
Super-Resolution from Joint Sparse Recovery

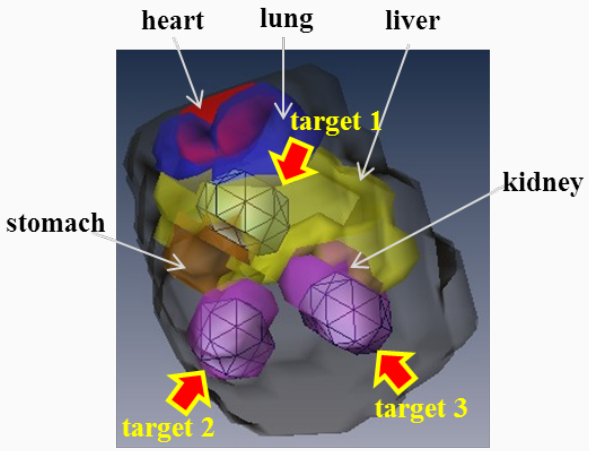
(Lim et al, OPEX, 2017)



Diffuse Optical Tomography Applications

(Lee, OPEX, 2013)

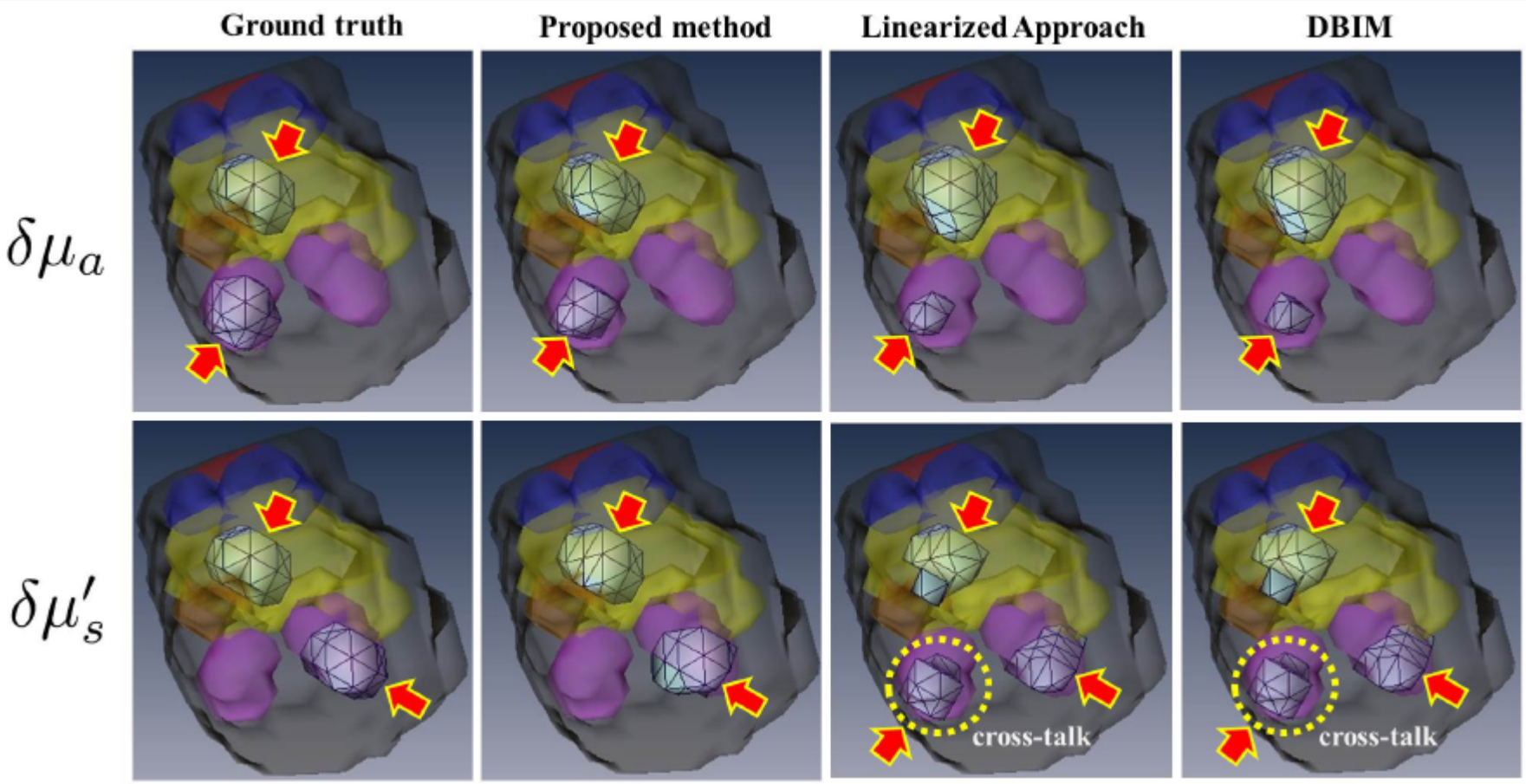




optical parameters in mm^{-1}	$\delta\mu_a$	$\delta\mu_s'$
Target 1	0.0504	0.8444
Target 2	0.0423	0.0
Target 3	0.0	0.8619

Diffuse Optical Tomography Applications

(Lee, OPEX, 2013)



Summary : Part I

- Sparsity principle is very important for biomedical image reconstruction
- Compressed sensing has many extensions
  - Compressed sensing
  - Motion compensated CS
  - Learning-based CS
  - MMV CS
- Downside: suffers from discretization, RIP, optimization bottleneck

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