SINGLE CHANNEL EXACT BLIND IMAGE DECONVOLUTION FROM RADIALLY SYMMETRIC FIR BLUR

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ABSTRACT

The multichannel exact blind image deconvolution theory tells us that the *exact* image reconstruction is possible without a prior knowledge of point spread function (PSF) as long as we can measure at least three blur images through distinct blur channels. However, in many biological applications, there exist many technical difficulties in applying the theory since the image content may vary due to various reasons such as the specimen drift between snapshots, or specimen damage due to prolonged exposure, or physiological changes in live cell imaging.

The main contribution of this paper is a new exact blind deconvolution theory that eliminates the need of multiple blur measurements, but still guarantees the *exact* reconstruction. The basic idea of such breakthrough is to exploit the *radial symmetry* of a certain class of PSFs. This makes the PSF estimation problem as 1-D channel identification problem with multiple excitations, which can be solved using subspace methods. Since radially symmetric PSFs can be quite often encountered in many practical applications such as optical imaging systems, and astigmatism corrected electron microscopy, our theory may have great impacts in many practical imaging applications.

1. INTRODUCTION

Many image acquisition process can be modeled as a two-dimensional (2-D) convolution of the true image with a point-spread function (PSF) often called as blur function. If the PSF is known *a priori*, there are several well known techniques for image restoration, such as Wiener filtering, iterative reconstruction, and so on (see the excellent review of [1] and the references therein). However, in practice, the PSF is difficult to measure, and only its partial information is available [1]. In this case, we need to estimate both PSF and the unknown image. This problem is termed the blind deconvolution. There exists several blind deconvolution methods which can be categorized into parametric and non-parametric approaches. The parametric approaches for optical microscopy or electron microscopy exploit the parametric decomposition of the OTF functions [1], or the location of frequency nulls to estimate the overall OTF functions [2]. Non-parametric approach usually employs the nonlinear iterative optimization techniques, in which the unknown PSF and the unknown images are alternatively estimated during the iteration steps. Classical Lucy-Richardson, Maximum-likelihood (ML), and the generalized cross-validation (GCV) approaches are among this class [1].

Recently, a revolutionary multichannel blind deconvolution methods [3] have been proposed which guarantees the exact reconstruction of the unknown PSF and image from multiple blurred images through a number of distinct blur channels. Harikumar and Bresler [3] showed that the exact estimation of the unknown PSF functions is almost surely guaranteed if there are at least three distinct measurements of the identical image through distinct blur channels. Unlike other blind deconvolution methods, the multichannel blind deconvolution approach does not require the unknown PSF to have frequency nulls or parametric decomposition; furthermore, the algorithm is in principle non-iterative.

However, there are many situations in which it is impossible to obtain multiple blur measurements of the identical images. For example, in electron-microscopy the specimen are prone to damage from prolonged exposure, and often drift during the multiple defocus setting. In fluorescent microscopy imaging, implementing the multiple blur kernel is basically impossible without additional apparatus such as structured illumination and etc. Moreover, between the multiple acquisition the fluorescent intensity could vary due to fast physiological changes.

The main contribution of this paper is a new exact blind deconvolution theory that eliminates the need of multiple blur measurements, but still guarantees the *exact* reconstruction. The basic idea of such breakthrough is to exploit the observation that the most PSF is *radially symmetric* in Fourier domain. This makes the PSF estimation problem as 1-D channel identification problem with multiple excitations, which can be solved using subspace methods. Since radially symmetric PSFs can be quite often encountered in many practical applications such as optical imaging systems, and electron microscopy with minimal astigmatism, our theory may have a

This work was supported by the Basic Research Program of the Korea Science and Engineering Foundation under Grant R01-2005-000-10035-0.
Note: This work has been filed for patent application. Reviewers are asked to keep confidential during review process.

great impact in many practical imaging applications. Simulation results also confirm our theory.

2. MEASUREMENT THROUGH RADIALLY SYMMETRIC BLUR

For a shift invariant point spread function (PSF), an observed noisy image $y(\mathbf{r})$ through a blur channel is described by convolution:

$$y(\mathbf{r}) = \int h(\mathbf{r}-\mathbf{r}')x(\mathbf{r}') + w(\mathbf{r}) \quad (1)$$

where $\mathbf{r} = (x, y)$ and $\mathbf{r}' = (x', y')$ denote the 2-D coordinates, and $y(\mathbf{r}), h(\mathbf{r}), x(\mathbf{r})$ and $w(\mathbf{r})$ denote the observed noisy image, point spread function (or blur kernel), the true images, and the additive noise, respectively. The Fourier transform of the convolution can be represented as the multiplication in Fourier domain:

$$Y(\mathbf{k}) = H(\mathbf{k})X(\mathbf{k}) + W(\mathbf{k}) \quad (2)$$

where $Y(\mathbf{k}), H(\mathbf{k}), X(\mathbf{k})$ and $W(\mathbf{k})$ are the corresponding 2-D Fourier transform at the 2-D spatial frequency $\mathbf{k} = (k_x, k_y)$, respectively. On the polar coordinate, Eq. (2) can be converted into

$$Y(k,\Theta) = H(k,\Theta)X(k,\Theta) + W(k,\Theta) \quad (3)$$

where $k = ||\mathbf{k}|| = \sqrt{k_x^2 + k_y^2}$ and $\Theta = \tan^{-1}\frac{k_y}{k_x}$, respectively.

Many practical applications such as the camera, optical microscopy and the astigmatism corrected electron microscopy, have radially symmetric PSFs. If a PSF is radially symmetric, the optical transfer function (OTF) in the Fourier domain $H(\mathbf{k})$ is also radially symmetric. More specifically, $H(k,\Theta) = H(\Theta)$ and Eq. (3) becomes

$$Y(k,\Theta) = H(k)X(k,\Theta) + W(k,\Theta) \quad (4)$$

which implies that the deconvolution problem becomes separable 1-D deconvolution problem for each Θ. Therefore, the blind deconvolution problem now becomes the one dimensional problem of estimating the 1-D FIR filter $H(k)$ using multiple measurements for various Θ.

3. SUBSPACE BASED EXACT BLIND DECONVOLUTION

One dimensional blind deconvolution or identification problems have been extensively investigated in wireless communication community to deal with the multipath interference problem [4]. Classical approach is to use the pilot signal with training sequence by sacrificing the communication bandwidth. Blind approach has been investigate to deal with this issue, and so called subspace methods have been investigated extensively due to their efficiency over the conventional methods [4]. The key idea of the subspace method is that a measurement can be modeled as a vector in the signal subspace spanned by the filtering kernel. Since the signal subspace and the noise subspace are orthogonal to each other, filtering kernel is, therefore, simply estimated by exploiting the orthogonality [4]. Now, we will discuss how this method can be used for our approach.

Note that for a fixed Θ, Eq. (4) can be represented by one dimensional convolution:

$$y_\Theta = h * x_\Theta + w_\Theta\,. \quad (5)$$

In discrete implementation, we assume that the 1-D filter h is an anti-causal finite impulse response (FIR) filter given by

$$h = \begin{bmatrix} h_{-M} & \cdots & h_0 & \cdots & h_M \end{bmatrix}^T. \quad (6)$$

Furthermore, we assume that $\Theta = i\Delta\Theta$ is the i-th discretized position of Θ, where $\Delta\Theta$ is the step size and $i = 0, \cdots, P-1$. In matrix form, Eq. (5) is then given by

$$\mathbf{y}_i = \mathbf{H}\mathbf{x}_i + \mathbf{w}_i, \;\; i = 0, \cdots, P-1 \quad (7)$$

where $\mathbf{y}_i$ and $\mathbf{x}_i$ are $(N+2M)\times 1$ measurement vector and $N \times 1$ the true signal vectors, respectively, which are given by:

$$\mathbf{y}_i = \begin{bmatrix} y_{-M}^i & \cdots & y_0^i & \cdots & y_{N+M-1}^i \end{bmatrix}^T \quad (8)$$

$$\mathbf{x}_i = \begin{bmatrix} x_0^i & \cdots & x_0^i & \cdots & x_{N-1}^i \end{bmatrix}^T \quad (9)$$

and the convolution matrix $\mathbf{H} \in \mathbb{C}^{(N+2M)\times N}$ is given by

$$\mathbf{H} = \begin{bmatrix} h_{-M} & \cdots & 0 \\ \vdots & \ddots & 0 \\ h_M & \ddots & h_{-M} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_M \end{bmatrix} \in \mathbb{C}^{(N+2M)\times N} \quad (10)$$

Assuming that the noise is independent of the true signal with variance σ_n^2, the correlation matrix of the observed signal $\mathbf{R_y}$ is given by

$$\mathbf{R_y} = \mathbf{H}\mathbf{R_x}\mathbf{H}^H + \sigma_n^2\mathbf{I} \quad (11)$$

where the empirical correlation matrix can be calculated by angular averaging: i.e. $\mathbf{R_y} = \frac{1}{P}\sum_i \mathbf{y}_i\mathbf{y}_i^H$ and $\mathbf{R_x} = \frac{1}{P}\sum_i \mathbf{x}_i\mathbf{x}_i^H$.

If $\mathbf{R_x}$ is full-ranked, the subspace of $\mathbf{Y}$ can be decomposed by singular value decomposition (SVD). Let $\lambda_0 \geq \lambda_1 \cdots \geq \lambda_{N+2M-1}$ be eigenvalues of $\mathbf{R_y}$. Then, the signal subspace is spanned by the eigenvectors corresponding to eigenvalues bigger than σ_n^2; and the remaining eigenvectors span the noise subspace. Under noiseless scenario, if the convolution matrix is full-column ranked, the eigenvectors corresponding to

$\lambda_0 \geq \lambda_1 \cdots \geq \lambda_{N-1}$ are identified as the set spanning the signal subspace. Let $\mathbf{S}$ and $\mathbf{G}$ denote the signal and the noise subspace, respectively:

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_0 & \mathbf{s}_1 & \cdots & \mathbf{s}_{N-1} \end{bmatrix} \quad (12)$$
$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 & \mathbf{g}_1 & \cdots & \mathbf{g}_{2M-1} \end{bmatrix} \quad (13)$$

where $\mathbf{S}_i$ and $\mathbf{G}_i$ denote the eigenvectors of the signal and the noise subspace, respectively.

Since the noise and the signal subspace are orthogonal to each other, any vectors in the noise space are orthogonal to the columns of the convolution matrix:

$$\mathbf{g}_i^H \mathbf{H} = \mathbf{0}, \;\; i = 0, \cdots, 2M-1 \,. \quad (14)$$

Therefore, it is possible to construct a quadratic cost function such that for the true convolution matrix the value of cost function will be zero. More specifically, the cost function is given by

$$c(\mathbf{H}) = \sum_{i=0}^{2M-1} |\mathbf{g}_i^H \mathbf{H}|^2 = \sum_{i=0}^{2M-1} \mathbf{g}_i^H \mathbf{H}\mathbf{H}^H \mathbf{g}_i \quad (15)$$

and our optimization problem is to find the $\mathbf{H}$ convolution matrix to minimize $c(\mathbf{H})$.

There is an interesting property for a convolution matrix to simplify the cost function. More specifically, we have

$$\mathbf{g}_i^H \mathbf{H} = \mathbf{h}^H \boldsymbol{\mathcal{G}}_i \quad (16)$$

where $\mathbf{h}$ denotes the FIR filter given by Eq. (6) and $\mathbf{g}_i = [g_0^i, \; g_1^i, \; \cdots, g_{N+2M-1}^i]^T$ and the matrix $\boldsymbol{\mathcal{G}}_i$ can be calculated as follows:

$$\boldsymbol{\mathcal{G}}_i = \begin{bmatrix} g_0^i & g_1^i & \cdots & g_{N-1}^i \\ g_1^i & g_2^i & \cdots & g_N^i \\ \vdots & \vdots & \ddots & \vdots \\ g_{2M}^i & g_{2M-1}^i & \cdots & g_{N+2M-1}^i \end{bmatrix} \quad (17)$$

Furthermore, if the blur kernel $\mathbf{h}$ has even symmetry, we can further impose the symmetry condition such that

$$\mathbf{g}_i^H \mathbf{H} = \mathbf{h}^H \boldsymbol{\mathcal{G}}_i^e \quad (18)$$

where $\boldsymbol{\mathcal{G}}_i^e = (\boldsymbol{\mathcal{G}}_i + \boldsymbol{\mathcal{G}}_i')/2$ and $\boldsymbol{\mathcal{G}}_i'$ are constructed by reordering $\boldsymbol{\mathcal{G}}_i$ from the last to the first rows. In summary, the simplified form of the cost function we need to minimize is given by

$$c(\mathbf{h}) = \mathbf{h}^T \mathbf{Q} \mathbf{h} \quad (19)$$

where $\mathbf{Q} = \left(\sum_{i=0}^{2M-1} \boldsymbol{\mathcal{G}}_i^e \, \boldsymbol{\mathcal{G}}_i^{eH}\right)$. Finding $\mathbf{h}$ under $||\mathbf{h}|| = 1$ corresponds to find the minimum eigenvector of $\mathbf{Q}$.

There is another way to define the cost function in terms of the signal subspace. Both cases give identical solutions. However, usually the signal-based estimation is robust to noise. Specifically, the cost function is given by

$$c(\mathbf{h}) = N|\mathbf{h}|^2 - \sum_{i=0}^{N-1} \mathbf{S}_i^H \mathbf{H}\mathbf{H}^H \mathbf{S}_i = N|\mathbf{h}|^2 - \mathbf{h}^H \tilde{\mathbf{Q}} \mathbf{h} \quad (20)$$

where $\tilde{\mathbf{Q}} = \left(\sum_{i=0}^{2M-1} \boldsymbol{\mathcal{S}}_i^e \, \boldsymbol{\mathcal{S}}_i^{eH}\right)$ and $\boldsymbol{\mathcal{S}}_i^e = (\boldsymbol{\mathcal{S}}_i + \boldsymbol{\mathcal{S}}_i')/2$ and $\boldsymbol{\mathcal{S}}_i$ are constructed using $\mathbf{s}_i$ similar to Eq. (17). Finding $\mathbf{h}$ to minimize Eq. (20) can be easily implemented by obtaining the eigenvector corresponding to the maximum eigenvalues of $\tilde{\mathbf{Q}}$.

The main strength of the subspace method is that the solution is the *exact* solution of the blind deconvolution problem. Specifically, the estimated filter $\tilde{\mathbf{h}}$ only differs by constant scaling.

4. IMPLEMENTATION ISSUES

In order to implement the blind deconvolution methods, we first need to obtain the $\mathbf{y}_i, i = 0, \cdots, P-1$. Basically, $\mathbf{y}_i$ can be obtained using the inverse Fourier transform of the $Y(k, i\Delta\Theta)$ along k direction. However, direct Fourier transform of an image to obtain the radial lines in Fourier space is prone to error since the discrete Fourier transform (DFT) approximation of the continuous time Fourier transform (CTFT) assumes the periodic repetition of the images, which breaks the radial symmetry of the OTFs.

In order to deal with this issue, we implement all the steps of the algorithm in radon space without using the DFT. This is owing to the powerful Fourier slice theory [2] saying that the one dimensional Fourier transform along radial direction corresponds the the radial slice of the 2-D continuous Fourier transform. Hence, the radon data itself corresponds to $\mathbf{y}_i, i = 0, \cdots, P-i$. In brief, the estimation of PSF and the image restoration process can be summarized as follows:

1. Apply the radon transform of the blurred image at every $\Delta\Phi$ angle to obtain the blurred sinogram $\mathbf{y}_i, i = 0, \cdots, P-1$.
2. The filter kernel $\mathbf{h}$ is estimated using the singoram by minimizing Eq. (20).
3. Obtain the restored sinogram $\mathbf{x}_i$ by solving matrix equation Eq. (7) using the least squared approach.
4. Obtained the final restored image using filtered backprojection of the restored sinogram data.

5. NUMERICAL RESULTS

The original image used for simulation is given in Fig. 1. We consider two radially symmetric PSF; a circular symmetric Gaussian filer and contrast transfer function of astigmatism corrected electron microscopes (see their frequency response in Fig. 2(a)(b), respectively).

Fig. 1. Original image for simulation study.

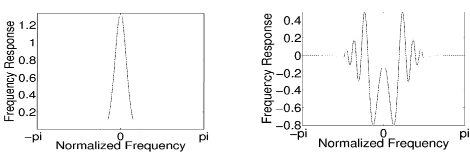
The OTF $H(k)$ of astigmatism corrected electron microscope is given by $H(k) = CTF(k)E(k)$ where the envelop function is defined by $E(k) = e^{-Bk^2}$ for some constant B, and the contrast transfer function $CTF(k)$ is given by

$$CTF(k) = -sin(\gamma(k) + \psi) \quad (21)$$

$$\gamma(k) = -\frac{\pi}{2}\left(C_s\lambda^3k^4 - 2\Delta z\lambda k^2\right) \quad (22)$$

$$\psi = \tan^{-1}(Q/\sqrt{1-Q^2}) \quad (23)$$

where $C_s, \lambda, \Delta z$ and Q denote the spherical aberration constant, electron wavelength, defocus, amplitude constant, respectively [2].



(a) (b)

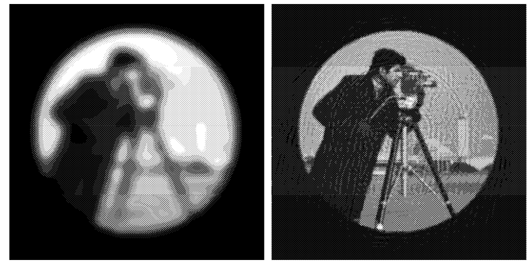
Fig. 2. OTF function for (a) a Gaussian blur, and (b) an astigmatism corrected electron microscope.

The blurred image by 2-D Gaussian filter and the restored image are given in Fig. 3(a)(b), respectively. In order to deal with the boundary effects during the radon transform and the restoration process, center circular region of the images are used and illustrated. Nearly perfect reconstruction was obtained using the proposed methods. Considering that the main lobe of Gaussian blur is mainly concentrated on low frequency range as shown in Fig. 2(a), only minimal reconstruction artifacts observed in Fig. 3(b) is impressive.

Similar results can be obtained for the blurred image from the astigmatism corrected electron microscope as illustrated in Fig. 4(a)(b). Nearly perfect reconstruction is obtained. Compared to the Gaussian, there exists multiples isolated zeros in CTF. However, the information loss due to the CTF was not significant since the frequency nulls are located at the isolated frequencies.

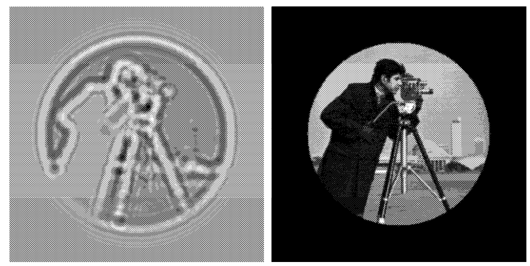
6. CONCLUSION

In this paper, we proposed a new exact blind deconvolution algorithm for radially symmetric blur kernels. Compared to the existing multichannel exact blind deconvolution methods, our approach is unique since only single blurred image is required and the estimation process is basically one dimensional. Our algorithm is fully implemented in sinogram space. Simulation results showed the outstanding performance of our algorithm.



(a) (b)

Fig. 3. (a) Blurred image using radially symmetic Gaussian PSF, and (b) restored image using the proposed method.



(a) (b)

Fig. 4. (a) Blurred image from astigmatism corrected electron microscope, and (b) restored image using the proposed method.

7. REFERENCES

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