MR artifacts removal using sparse + low rank decomposition of annihilating filter based Hankel matrix

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Synopsis

In this paper, we propose a sparse and low-rank decomposition of annihilating filter-based Hankel matrix for removing MR artifacts such as motion, RF noises, or herringbone artifacts. Based on the observation that some MR artifacts are originated from k-space outliers, we employ a recently proposed image modeling method using annihilating filter-based low-rank Hankel matrix approach (ALOHA) to decompose the sparse outliers from the low-rank component. The proposed approach can be applied even for static images, because the k-space low rank component comes from the intrinsic image properties. We demonstrate that the proposed algorithm clearly removes several types of artifacts such as impulse noises, motion artifacts, and herringbone artifacts.

Purpose

Many types of MRI artifacts are originated from outliers in k-space measurements. For example, herringbone artifacts are scattered all over the image, which make the images unusable. The origin of this artifact is from electromagnetic spikes by gradient coils or fluctuating power supply which usually results in outliers in k-space domain. Motion artifacts are also commonly observed when a subject moves during the scanning. Due to the movement during k-space scanning, some of the k-space data lose their integrity from the previous k-space data, which can be also considered as outliers. In this paper, we propose a sparse and low-rank decomposition algorithm for removal of MRI artifacts. By utilizing the recently proposed annihilating filter-based low rank Hankel matrix (ALOHA) approach for image modeling [1,2], this paper shows that many MR artifacts can be decomposed into sparse outliers whereas k-space data from the underlying artifact-free image can be decomposed as a low-rank component of Hankel matrix.

Method

Recently, ALOHA was proposed as a powerful tool for compressed sensing MRI [1] and image processing [2]. More specifically, if the signal $$$x(\mathbf{r})$$$ is sparse, then there exists an annihilating function $$$h(\mathbf{r})$$$ such that $$x(\mathbf{r}) h(\mathbf{r}) = 0 \overset{FT}{\longleftrightarrow} \widehat{x}(\mathbf{k}) * \widehat{h}(\mathbf{k}) = 0,\quad(1)$$ where $$$\widehat{x}(\mathbf{k})$$$ denotes the Fourier

transform of $$$x(\mathbf{r})$$$. In particular, if $$$x(\mathbf{r})$$$ is represented as sum of Diracs, then $$$\widehat{h}(\mathbf{k})$$$ becomes a finite impulse response filter [1,3]. Thus, Eq. (1) can be rewritten as matrix-vector form: $$\mathscr{H}\{\widehat{x}\} \mathbf{h} = 0,\quad(2)$$ where $$$\mathscr{H}\{\cdot\}$$$ is Hankel matrix and $$$\mathbf{h}$$$ is the reverse ordered, vectorized annihilating filter. From this equation, we observe that Hankel matrix $$$\mathscr{H} \{\widehat{x}\}$$$ is low-ranked. This model can be generalized to signals that can be sparsified in a transform domain. If the image can be sparsified using wavelets whose spectrum is given by $$$\widehat \psi(\mathbf{k})$$$, then the low-rank Hankel matrix can be constructed for each band using weighted k-space data with wavelet weighting [1]. As mentioned before, some artifacts in MRI can be modeled as k-space sparse outliers. Accordingly, MRI k-space measurements $$$\widehat y(\mathbf{k})$$$ can be modeled as $$ \widehat{y}(\mathbf{k}) = \widehat{x}(\mathbf{k}) + \widehat{\epsilon}(\mathbf{k}),\quad(3)$$ where $$$\widehat{x}(\mathbf{k})$$$ is a k-space data of artifact-free image and $$$\widehat{\epsilon}(\mathbf{k})$$$ is sparse k-space outlier. In Eq. (3), if the underlying image is sparse, the first term has an annihilation property as reviewed in Eq. (1)-(2), whereas the second term is irrelevant with annihilation property because of irregular structures. This implies that we can use the annihilation property as a differentiation tool between MRI artifacts and true MR images. Thus, if we perform a lifting to Hankel structured matrix, then, we have $$ \mathscr{H}\{\widehat{\mathbf{Y}}\odot\widehat{\mathbf{W}}\}=\underbrace{\mathscr{H}\{\widehat{\mathbf{X}}\odot\widehat{\mathbf{W}}\}}_{\mbox{low-rank}}+\underbrace{\mathscr{H}\{\widehat{\mathbf{E}}\odot\widehat{\mathbf{W}}\}}_{\mbox{sparse outlier}},\quad(4) $$ where $$$\odot$$$ is Hadamard product, and $$$\widehat{\mathbf{X}}$$$ and $$$\widehat{\mathbf{W}}$$$ denote the matrices constructed from discretized samples of $$$\widehat x(\mathbf{k})$$$ and $$$\widehat \psi(\mathbf{k})$$$, respectively. Note that lifted Hankel matrix from sparse components is still sparse as shown in Fig. 1. Therefore, Eq. (4) becomes a structure for sparse + low-rank decomposition. Because RPCA (robust principal component analysis) [4] has been extensively investigated for sparse + low-rank decomposition, we employ the main idea of RPCA to decompose ALOHA matrix for a removal of MRI artifact. The proposed sparse + low-rank decomposition of Hankel matrix was implemented using SVD-free ADMM [1,2] as successfully demonstrated for impulse noise removal in images [5].

Results

We demonstrated three types of artifacts: impulse noises in k-space, random motion artifacts, and herringbone artifacts. For the impulse noise, we generated retrospective k-space measurements from a real in-vivo brain data (Siemens Verio 3T; 2D SE sequence; TR/TE=4000/100ms; 256^2 matrix; FOV=240^2 mm^2). We added impulse k-space noises for x5 downsampled k-space measurements. Fig. 2. showed that the artifacts were clearly removed from the images using the proposed method. For motion artifacts, 2D Cartesian GRE sequence was used for chest imaging with abrupt motion along the phase encoding direction (Siemens Trio Tim 3T; TR/TE=15/3.61ms; 256^2 matrix; FOV=500^2 mm^2). As observed in Fig. 3, we successfully decomposed image from motion artifacts. For Herringbone artifacts, we obtained an image from [6]. As observed in Fig. 4, we again decomposed true image as a low-rank component. As expected, spectral components from the artifacts were sparse in spectrum domain.

Conclusion

We proposed a novel sparse and low-rank decomposition method for correcting k-space MR artifacts such as motion or random glitch during acquisition. The proposed algorithm successfully decomposed sparse outliers from weighted Hankel matrix in k-space. The main principle behind the proposed algorithm is that the underlying artifact-free image can be sparsified in a transform domain, so we can decompose a low-rank Hankel matrix data out of corrupted k-space.

Acknowledgements

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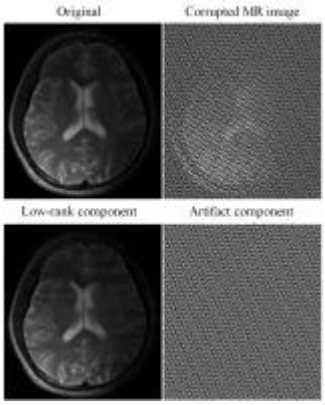
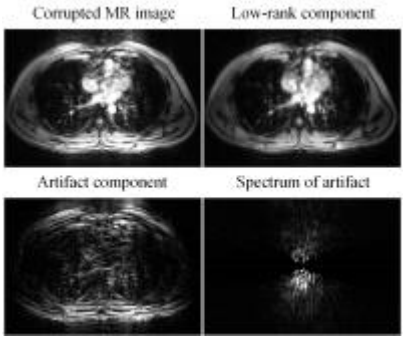
6. Case courtesy of Dr Roberto Schubert, Radiopaedia.org, rID: 16743.

Figures

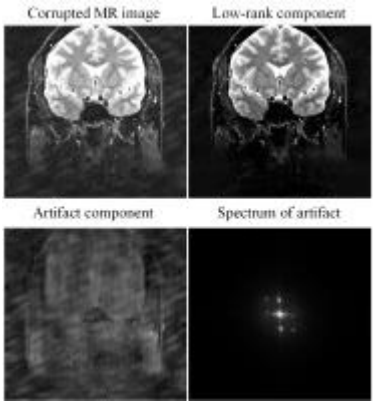
Lifting of MR k-space measurements with artifacts to a Hankel matrix composed of low-rank and sparse components.



Retrospective reconstruction images and artifacts from five-fold accelerated k-space samples corrupted by impulsive noises.



Reconstruction results of in-vivo motion artifacts.



Reconstruction results of herringbone artifacts.

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