Joint Sparse Recovery in Inverse Scattering

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ABSTRACT

In this paper, we review joint sparse recovery based reconstruction approach for inverse scattering problems that can solve the nonlnear inverse scattering probleme without linearization or iterative Green's function update. The main idea is to exploit the common support conditions of anomalies during multiple illumination or current injections, after which unknown potential or field can be estimated using recursive integral equation relationship. Explicit derivation for electric impedance tomography and diffuse optical tomography are discussed.

Keywords: Inverse scattering, joint sparse recovery, electric impedance tomography, diffuse optical tomography, compressed sensing

1. INTRODUCTION

Scattering theory, which is one of the central themes in modern mathematical physics, is mainly concerned with the effect that an inhomogeneous medium has on an incident photons or waves. In particular, if there exists inhomogeneities within a homogeneous medium, then the wave (or photon) will be perturbed or scattered. The direct scattering problem is to determine the scattered field from the knowledge of the homogeneous field and the inhomogeneous medium. On the other hand, inverse scattering takes this answers to the direct scattering problem as a starting point to retrieve the unknown inhomogeneous medium from the measured scattered fields.

Inverse scattering has many biomedical applications such as ultrasound, diffuse optical tomography (DOT), electric impedance tomography (EIT), and etc. However, a major difficulty in inverse scattering problems is that the imaging problem is highly nonlinear due to the nonlinear coupling between the unknown parameters and the fields. Furthermore, the problem is severely ill-posed due to the compactness of the forward operator.

Conventionally, there have been two types of inversion approaches to deal with the nonlinearity: linearization pproach and iterative methods. Born or Rytov approximation are typically used for linearisation approach, and the resulting linear integral equation can be usually solved using analytic approaches such as Fourier or Fourier-Laplace transform. Usually the computation times for these types of algorithms are within a few minutes even for a large 3-D phantom. However, the linearization approaches fail when the contrast between the homogeneous background and the unknown target is beyond the Born approximation limit. On the other hand, the iterative methods require re-calculating the unknown flux during each iteration. However, even with these high performance algorithms, the computational overhead of the nonlinear iterative algorithms is still prohibitive, especially for 3-D imaging, since Green's function needs to be recalculated at each iteration.

In this paper, we will review our recent approach that direct solves the non-linear inverse scattering problem by exploiting the joint sparsity of inhomogeneity distribution.[1,2] In this approach, no linearization is made, and iterative calculation of Green's function is not necessary. However, still accurate estimation of both unknown field and the inhomogeneities is possible thank to the recursive relationship of the forward integral equation. While the approach is quite general and can be used for many different type of inverse scattering problems, here we first review the joint sparse recovery approach for inverse scattering problem in EIT, then application to DOT will be discussed.

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Image Reconstruction from Incomplete Data VIII, edited by Philip J. Bones, Michael A. Fiddy, Rick P. Millane,
Proc. of SPIE Vol. 9600, 96000C · © 2015 SPIE · CCC code: 0277-786X/15/$18 · doi: 10.1117/12.2188764

2. FORWARD MODEL

2.1 Electric Impedance Tomography

Electrical impedance tomography (EIT) is a noninvasive imaging technique to reconstruct the electrical property of a medium based on the boundary measurement of the voltages that result from the injected currents. With the help of relatively low-cost imaging equipment and the fact that various materials such as biological tissues have their own conductivity values,[3] EIT has been applied for various clinical and industrial applications such as monitoring internal organs in human body.[4–6]

The electrical properties of a material are characterized by

$$\tilde{\sigma} = \sigma + j\omega\varepsilon, \tag{1}$$

where σ is the electric conductivity, ω the angular frequency of the applied current waveform, and ε the electric permittivity. In this paper, we deal with the static problem ($\omega = 0$) for simplicity. Let Ω be a bounded domain in $\mathbb{R}^d$, $d = 2, 3$, which is occupied by the homogeneous material of the conductivity 1. We suppose that a finite number of isotropic anomalies are embedded in the background domain Ω. We denote the anomalies by $D_1, \ldots, D_N$ and the corresponding conductivities by $\sigma_1, \ldots, \sigma_N$ where N is the number of anomalies. So the conductivity profile of the domain Ω is given by

$$\sigma = \chi(\Omega \setminus \cup_{p=1}^{N} D_p) + \sum_{p=1}^{N} \sigma_p \chi(D_p), \tag{2}$$

where the symbol $\chi(D)$ indicates the characteristic function of D. We let σ_p's be smooth and satisfy the ellipticity condition

$$0 < c \leq \sigma_p(x) \leq C < \infty \quad \text{for } x \in D_p, \quad p = 1, \ldots, N$$

for some positive constants c and C.

The EIT problem we consider in this paper is to find the locations, geometric features, and conductivities of anomalies using a finite number of pairs of voltages (Dirichlet data) and currents (Neumann data) on the boundary of Ω. Let $g_1, \ldots, g_M \in L_0^2(\partial\Omega)$ be the M number of given currents on $\partial\Omega$. Here $L_0^2(\partial\Omega)$ means the set of square integrable functions defined on $\partial\Omega$ with zero means. The currents g_m's are given functions. The corresponding internal potential u_m in Ω for $1 \leq k \leq M$ satisfies the Neumann boundary value problem

$$\begin{cases} \nabla \cdot (\sigma(x)\nabla u_m) = 0 \quad \text{in } \Omega, \\ \dfrac{\partial u_m}{\partial \nu}\Big|_{\partial\Omega} = g_m, \\ \displaystyle\int_{\partial\Omega} u_m \, d\sigma = 0. \end{cases} \tag{3}$$

We tackle the reconstruction problem of the anomalies D_p's and their conductivities σ_p's based on an integral representation formula of u_m. We derive the formula in the remaining of this section.

2.1.1 Integral Represenation

The Neumann function $N(\cdot, y)$ on Ω is the solution to

$$\begin{cases} -\Delta_x N(x, y) = \delta_y(x) \qquad \text{in } \Omega, \\ \dfrac{\partial}{\partial \nu_x} N(x, y)\Big|_{\partial\Omega} = -\dfrac{1}{|\partial\Omega|}, \\ \displaystyle\int_{\partial\Omega} N(x, y) \, d\sigma(x) = 0 \end{cases} \tag{4}$$

for $y \in \Omega$. Let U_m be the electric potential in absence of anomalies, *i.e.*, the solution to

$$\begin{cases} \Delta U_m = 0 \quad \text{in } \Omega, \\ \left.\dfrac{\partial U_m}{\partial \nu}\right|_{\partial\Omega} = g_m, \\ \displaystyle\int_{\partial\Omega} U_m \, d\sigma = 0. \end{cases} \tag{5}$$

Then, U_m can be represented as

$$U_m(x) = \int_{\partial\Omega} N(x,y) g_m(y) \, d\sigma(y), \quad x \in \Omega.$$

Note that because of the third condition in (3) and the second equation in (4) we have

$$\int_{\partial\Omega} \frac{\partial}{\partial \nu_y} N(x,y) u_m(y) \, d\sigma(y) = 0, \quad x \in \Omega.$$

Thus we have

$$U_m(x) = \int_{\partial\Omega} \left[N(x,y) \frac{\partial u_m}{\partial \nu}(y) - \frac{\partial}{\partial \nu_y} N(x,y) u_m(y) \right] d\sigma(y).$$

We then have from the Green's identity

$$\begin{aligned} U_m(x) = &- \int_{\Omega \setminus \cup_{p=1}^N D_p} \Delta_y N(x,y) u_m(y) \, dy \\ &+ \sum_{p=1}^N \int_{\partial D_p} \left[N(x,y) \left.\frac{\partial u_m}{\partial \nu}\right|_+ (y) - \frac{\partial}{\partial \nu_y} N(x,y) u_m(y) \right] d\sigma(y). \end{aligned}$$

It then follows from the transmission conditions (continuity of flux and potential) of u_m along ∂D_p's that

$$u_m(x) - U_m(x) = \int_{\cup_{p=1}^N D_p} (1 - \sigma(y)) \nabla_y N(x,y) \cdot \nabla u_m(y) \, dy, \quad x \in \overline{\Omega}. \tag{6}$$

2.2 Diffuse Optical Tomgoraphy

The near-infrared (NIR) optical wavelength range ($700 \sim 1000$nm) provides an important opportunity for biological imaging, as tissues are relatively transparent in this regime due to the low absorption rate of the primary absorbers.[7,8] It is now well-known that NIR light can penetrate tissue up to a depth of several centimeters. Accordingly, the objective of DOT is to exploit this window of opportunity to reconstruct the optical properties of highly scattering biological tissue from scattered and attenuated optical flux measured at the boundary of the medium.[9–11] Extensive studies have been conducted for the theoretical modeling of photon propagation in tissue and to realize accurate reconstructions of the optical parameters.[12,13]

Let Ω be a bounded domain in $\mathbb{R}^d$, $d = 2, 3$, which is occupied by the background materials with known optical parameters. We suppose that a finite number of isotropic anomalies are embedded in the background domain Ω, where $D_1, \ldots, D_N$ denote the abnormalies. In DOT, the scattering and absorption are the main parameters of our interest. Let μ and $d = 1/3(s' + \mu)$ are the absorption and diffusion coefficients, where s' denotes the reduced scattering coefficient. Then, optical parameters are distributed by

$$\begin{cases} d_\delta = d_0 \chi(\Omega \setminus \cup_{p=1}^N D_p) + \displaystyle\sum_{p=1}^N d_p \chi(D_p) \\ \mu_\delta = \mu_0 \chi(\Omega \setminus \cup_{p=1}^N D_p) + \displaystyle\sum_{p=1}^N \mu_p \chi(D_p) \end{cases} \tag{7}$$

where the symbol $\chi(D)$ indicates the characteristic function of D.

Photon migration within biological tissues is usually modeled by the transport equation. However, when light scattering prevails over absorption, the propagation of light can be modeled by a diffusion equation:

$$\begin{cases} \nabla \cdot d_\delta \nabla u_m - \mu_\delta u_m = 0, & \text{in } \Omega, \\ u_m + \ell \frac{\partial u_m}{\partial \nu} = g_m & \text{on } \partial\Omega \end{cases} \tag{8}$$

where ℓ is a parameter related to the diffusion coefficient, dimension and reflection on the boundary,[14] and g_m denotes the exciting photon flux from m-th source distribution at the boundary. In the absence of anomalies, the solution is given by

$$\begin{cases} \Delta U_m - k_0^2 U_m = 0 & \text{in } \Omega, \\ U_m + \ell \frac{\partial U_m}{\partial \nu} = g_m & \text{on } \partial\Omega \end{cases} \tag{9}$$

where $k_0^2 = \mu_0/d_0$. Suppose, furthermore, that $G_0(x,y)$ be the Dirichlet Green function for the homogeneous medium without anomalies, i.e. for each $y \in \Omega$, G_0 is the solution of

$$\begin{cases} \Delta G_{m_0}(x,y) - k_0^2 G_{m_0}(x,y) = -\delta_y(x), & \text{in } \Omega, \\ G_{m_0}(x,y) + \ell \frac{\partial G_{m_0}(x,y)}{\partial \nu} = 0 & \text{on } \partial\Omega \end{cases} \tag{10}$$

Then U_m can be represented as

$$U_m(x) = \frac{1}{\ell} \int_{\partial\Omega} G_{m_0}(x,y) g_m(y)\ d\sigma(y), \quad x \in \Omega.$$

Note that because of the second condition in (8) and the second equation in (10) we have

$$\frac{1}{\ell} \int_{\partial\Omega} \left(G_{m_0}(x,y) + \ell \frac{\partial G_{m_0}(x,y)}{\partial \nu} \right) u_m(y)\ d\sigma(y) = 0, \quad x \in \Omega.$$

Thus we have

$$\begin{aligned} U_m(x) &= \frac{1}{\ell} \int_{\partial\Omega} G_{m_0}(x,y) g_m(y)\ d\sigma(y) - \frac{1}{\ell} \int_{\partial\Omega} \left(G_{m_0}(x,y) + \ell \frac{\partial G_{m_0}(x,y)}{\partial \nu} \right) u_m(y)\ d\sigma(y) \\ &= \int_{\partial\Omega} \left[G_{m_0}(x,y) \frac{\partial u_m}{\partial \nu}(y) - \frac{\partial}{\partial \nu_y} G_{m_0}(x,y) u_m(y) \right]\ d\sigma(y). \end{aligned} \tag{11}$$

We then have from the Green's identity

$$\begin{aligned} U_m(x) = &\int_{\Omega \setminus \cup_{p=1}^N D_p} \left[G_{m_0}(x,y) \Delta_y u_m(y) - \Delta_y G_{m_0}(x,y) u_m(y) \right]\ dy \\ &+ \sum_{p=1}^N \int_{\partial D_p} \left[G_{m_0}(x,y) \frac{\partial u_m}{\partial \nu}\Big|_+ (y) - \frac{\partial}{\partial \nu_y} G_{m_0}(x,y) u_m(y) \right]\ d\sigma(y). \end{aligned}$$

From the transmission conditions (continuity of flux and potential) of u_m along ∂D_p's such that

$$d_p \frac{\partial u_m}{\partial \nu}\Big|_- = d_0 \frac{\partial u_m}{\partial \nu}\Big|_+, \qquad u_m|_- = u_m|_+ \qquad \text{on } \partial D_p,\ p = 1, \cdots, N, \tag{12}$$

we obtain the following formula:

$$\begin{aligned} u_m(x) - U_m(x) &= \int_{\cup_{p=1}^N D_p} \left(\frac{d_0(y) - d_p(y)}{d_0(y)} \right) \nabla_y G_{m_0}(x,y) \cdot \nabla u_m(y)\ dy \\ &\quad + \int_{\cup_{p=1}^N D_p} \left(\frac{\mu_0(y) - \mu_p(y)}{d_0(y)} \right) G_{m_0}(x,y) u_m(y)\ dy, \quad x \in \overline{\Omega}. \end{aligned} \tag{13}$$

3. JOINT SPARSE RECOVERY FORMULATION

This section describes the non-iterative exact reconstruction method for EIT and DOT problems based on the joint sparsity recovery algorithm. It reconstructs the anomalies accurately without an iterative procedure or a linear approximation.

3.1 Electric Impedance Tomography

Let us first fix some notations. For the potential function u_m for $1 \le m \le M$, we define the current on the anomalies as

$$I_m(y) = \begin{cases} (1-\sigma_p(y))\nabla u_m(y) & \text{for } y \in D_p,\ p = 1, \dots, N, \\ 0 & \text{for } y \in \Omega \setminus \cup_{p=1}^N D_p. \end{cases} \tag{14}$$

Then the formula (6) becomes

$$u_m(x) - U_m(x) = \int_\Omega \nabla_y N(x-y) \cdot I_m(y)\, dy, \quad x \in \partial\Omega. \tag{15}$$

Remind that the anomalies $D_1, \dots, D_p$ are located at fixed positions despite of the different boundary conditions $g_1, \dots, g_M$, whereas the currents I_m on the anomalies vary. Therefore, assuming the sparsity for the support set $\cup_{p=1}^N D_p$, the problem (15) is a joint sparsity problem since the non-zero current location (the non-zero rows in the associated linear equation) is independent of the boundary currents (the given columns). To describe it more specifically, let us assume that I_m, $1 \le m \le M$, is approximated by either piecewise constant functions or splines:

$$I_m(y) = \begin{bmatrix} \sum_{j=1}^n I_{m,1}(y_{(j)})\, b_1(y, y_{(j)}) \\ \sum_{j=1}^n I_{m,2}(y_{(j)})\, b_2(y, y_{(j)}) \end{bmatrix}, \quad y \in \Omega, \tag{16}$$

where b_d, $d = 1, 2$, is the basis function for the d-th coordinate and $\{y_{(j)}\}_{j=1}^n$ is the finite discretization points of Ω.

After substituting (16) into (15), we have the following equation:

$$u_m(x) - U_m(x) = \sum_{d=1}^2 \sum_{j=1}^n \tilde{N}_d(x, y_{(j)})\, I_{m,d}(y_{(j)}), \quad x \in \partial\Omega, \tag{17}$$

where

$$\tilde{N}_d(x, y_{(j)}) = \int_\Omega \nabla_{y,d} N(x-y)\, b_d(y, y_{(j)})\, dy.$$

Here $\nabla_{y,d} N(x-y)$ $(d = 1, 2)$ means the d-th coordinate component of $\nabla_y N(x-y)$. We now define, respectively, the sensing matrix $A = [A_1, A_2] \in \mathbb{R}^{m \times 2n}$, the currents $X = [X_1^T, X_2^T]^T \in \mathbb{R}^{2n \times M}$ and the measurements $Y \in \mathbb{R}^{m \times M}$ as

$$\big(A_d\big)_{ij} = \tilde{\Gamma}_d(x_{(i)}, y_{(j)}), \quad \big(X_d\big)_{jk} = I_{m,d}(y_{(j)}), \quad \text{and } \big(Y\big)_{ik} = (u_m - U_m)(x_{(i)}),$$

where $\{x_{(i)}\}_{i=1}^m$ is the collection of the finite number of measurement locations on $\partial\Omega$. Then we can formulate (17) as the following matrix equation:

$$Y = AX = [A_1, A_2] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{18}$$

From (14), the solution X to (25) has a pairwise joint sparsity meaning that X_1 and X_2 are nonzero at the same rows which correspond to the positions where the anomalies are located. Based on this equation, we can formulate the following joint sparse recovery problem:[15]

$$\min_X \|X\|_0, \quad \text{subject to} \quad \|Y - AX\|_F \le \eta, \tag{19}$$

where $\|\cdot\|_F$ denotes the Frobenius norm and η denotes the noise level.

After having the solution X to (19) by applying a joint sparse recovery algorithm, one can estimate the anomaly positions by collecting $y_{(j)}$ whose corresponding currents $(X_d)_{jk}$ are nonzero for all k and d. Since this criterion reconstructs the points which belong to any one of D_p's, we denote the obtained anomaly positions by $\hat{D}$. With the solution X to (19) and $\hat{D}$, we have the current on the anomaly as well, say $\hat{I}_m(y_{(j)})$ for $y_{(j)} \in \hat{D}$. Then the unknown solution u_m can be now easily estimated using (**??**) as

$$\hat{u}_m(x) = U_m(x) + \int_{\hat{D}} \nabla_y N(x-y) \cdot \hat{I}_m(y)\, dy, \quad x \in \hat{D}. \tag{20}$$

Finally, the conductivity σ is calculated by solving

$$(u_m - U_m)(x) = \int_{\hat{D}} (\sigma(y)-1)\nabla_y N(x-y) \cdot \nabla \hat{u}_m(y)\, dy, \quad x \in \partial\Omega \tag{21}$$

for $1 \le k \le M$. Note that every term in (21) except $(\sigma - 1)$ is now known since the potential u_m, which was initially measured only on $\partial\Omega$, is estimated on the whole anomalies $\hat{D}$ from (20). We emphasize that (21) is a linear equation for $(\sigma(y_{(j)}) - 1)$. Hence neither linear approximation nor the iterative update is required. Furthermore, we can expect more efficient and less ill-posed reconstruction procedure due to the knowledge of the estimated position of anomalies.

3.2 Diffuse Optical Tomography

This section now describes the non-iterative exact reconstruction method for DOT problem based on the joint sparsity recovery algorithm. It reconstructs the anomalies accurately without an iterative procedure or a linear approximation.

Let us first fix some notations. For the potential function u_m for $1 \le m \le M$, we define the current on the anomalies as

$$I_m(y) = \begin{cases} \nabla \cdot r(y)\nabla u_m(y) - q(y)u_m(y) & \text{for } y \in D_p,\ p = 1, \dots, N, \\ 0 & \text{for } y \in \Omega \setminus \cup_{p=1}^N D_p. \end{cases} \tag{22}$$

where

$$r(y) := \left(\frac{d_0(y) - d_p(y)}{d_0(y)}\right),\ q(y) := -\left(\frac{\mu_0(y) - \mu_p(y)}{d_0(y)}\right).$$

After some manipulation using Green's formula, the formula (13) becomes

$$u_m^s(x) = \int_\Omega G_0(x,y) I_m(y)\, dy, \quad x \in \partial\Omega. \tag{23}$$

Now, let us assume that I_m, $1 \le m \le M$, is approximated by either piecewise constant functions or splines:

$$I_m(y) = \sum_{j=1}^n I_m(y_{(j)})\, b(y, y_{(j)}) \quad y \in \Omega, \tag{24}$$

where b is the basis function and $\{y_{(j)}\}_{j=1}^n$ is the finite discretization points of Ω. We define, respectively, the sensing matrix $A \in \mathbb{R}^{m\times n}$, the currents $X \in \mathbb{R}^{n\times M}$ and the measurements $Y \in \mathbb{R}^{m\times M}$ as

$$\big(A\big)_{ij} = G_0(x_{(i)}, y_{(j)}), \quad \big(X\big)_{jm} = I_m(y_{(j)}), \quad \text{and } \big(Y\big)_{im} = u_m^s(x_{(i)}),$$

where $\{x_{(i)}\}_{i=1}^m$ is the collection of the finite number of measurement locations on $\partial\Omega$. Then we can formulate (17) as the following matrix equation:

$$Y = AX \tag{25}$$

Then, the simultaneous reconstruction of the DOT problem from multiple illumination cases, where the support of the optical parameter variation is sparse, can be stated as the following joint sparse recovery problem:[15,16]

$$\min_X \ ||X||_0, \ \text{ subject to } \ ||Y - AX||_F \leq \epsilon. \tag{26}$$

Here, $||X||_F$ is the Frobenius norm of matrix X.

After having the solution X to (26) by applying a joint sparse recovery algorithm, one can estimate the anomaly positions by collecting $y_{(j)}$ whose corresponding currents $(X)_{jm}$ are nonzero for all m. Since this criterion reconstructs the points which belong to any one of D_p's, we denote the obtained anomaly positions by $\hat{D}$. With the solution X to (19) and $\hat{D}$, we have the current on the anomaly as well, say $\hat{I}_m(y_{(j)})$ for $y_{(j)} \in \hat{D}$. Then the unknown solution u_m can be now easily estimated as

$$\hat{u}_m(x) \quad = \quad U_m(x) + \int_{\hat{D}} G_{m_0}(x,y) I_m(y) dy, \quad x \in \overline{\Omega}. \tag{27}$$

Finally, we can estimate

$$\begin{aligned} u_m(x) - U_m(x) \quad &= \quad \int_{\hat{D}} r(y) \nabla_y G_{m_0}(x,y) \cdot \nabla \hat{u}_m(y) \ dy \\ &\quad + \int_{\hat{D}} q(y) G_{m_0}(x,y) \hat{u}_m(y) \ dy, \quad x \in \overline{\Omega}. \end{aligned} \tag{28}$$

for $1 \leq m \leq M$. Note that every term in (28) except $(q(y), r(y))$ is now known since the potential u_m, which was initially measured only on $\partial\Omega$, is estimated on the whole anomalies $\hat{D}$ from (20). We emphasize that (21) is a linear equation for $(q(y), r(y))$. Hence neither linear approximation nor the iterative update is required. Furthermore, we can expect more efficient and less ill-posed reconstruction procedure due to the knowledge of the estimated position of anomalies.

4. NUMERICAL RESULTS

For the numerical results in EIT and DOT, we refer our works.[1,2] The common observations from these experiments are as following:

- The quantitative error measures showed that the joint sparse recovery approach outperforms the linearized and non-linear iterative approaches. This is because the internal potential or fields can be estimated using the recursive integral representation, which decouples the nonlinearity from ill-posedness of inverse problems.
- The unknown internal flux originated from inhomogeneities were also accurately estimated.
- For simultaneous reconstruction of absorption and scattering parameters in DOT problem, the cross talk artifacts between absorption and scattering were significantly reduced even with the continuous wave (CW) modulation. This clearly showed that the cross talk artifact is due to the reconstruction algorithm, rather than intrinsic properties.
- Even though the joint sparse recovery algorithm is based on nonlinear model, the computational complexity was much lower than the linearization approaches. This is because the joint sparse recovery steps reduces the problem size, which reduces the overall computational time.

5. CONCLUSION

This paper reviews joint sparsity based reconstruction method that resolves the non-linearity of the inverse scattering problems such as EIT and DOT. It accurately reconstructs the anomalies without iterative procedure or linear approximation. The main idea of the proposed method comes from the joint sparsity in the compressed sensing theory. The non-linear inverse problem of an inverse scattering problem can be changed into the joint sparse recovery problem, and it enables us to estimate the unknown internal potential with the help of the recursive nature of the forward problem formulation. Finally, the property of the anomalies can be calculated from the proposed linear problem. The simulation results showed that the proposed method outperforms over the linearized and nonlinear approaches.

REFERENCES

1. O. K. Lee, H. Kang, J. C. Ye, and M. Lim, "A non-iterative method for the electrical impedance tomography based on joint sparse recovery," *Inverse Problems* **31**(7), p. 075002, 2015.
2. O. Lee and J. C. Ye, "Joint sparsity-driven non-iterative simultaneous reconstruction of absorption and scattering in diffuse optical tomography," *Optics express* **21**(22), pp. 26589–26604, 2013.
3. L. Borcea, "Electrical impedance tomography," *Inverse Problems* **18**, pp. 99–136, 2002.
4. G. Alessandrini and L. Rondi, "Stable determination of a crack in a planar inhomogeneous conductor," *SIAM J. Math. Anal.* **30**(2), pp. 326–340, 1998.
5. E. L. V. Costa, R. G. Lima, and M. B. P. Amato, "Electrical impedance tomography," *Curr. Opin. Crit. Care* **15**, pp. 18–24, 2009.
6. R. L. Parker, "The inverse problem of resistivity sounding," *Geophysics* **49**, pp. 2143–2158, December 1984.
7. A. Villringer and B. Chance, "Non-invasive optical spectroscopy and imaging of human brain function," *Trends Neurosci.* **20**, pp. 435–442, 1997.
8. D. A. Boas, D. H. Brooks, E. L. Miller, C. A. DiMarzio, M. Kilmer, R. J. Gaudette, and Q. Zhang, "Imaging the body with diffuse optical tomography," *IEEE Signal Process. Mag.* **18**, pp. 57–75, November 2001.
9. D. A. Benaron, S. R. Hintz, A. Villringer, D. Boas, A. Kleinschmidt, J. Frahm, C. Hirth, H. Obrig, J. C. van Houten, E. L. Kermit, W. F. Cheong, and D. K. Stevenson, "Noninvasive functional imaging of human brain using light," *J. Cereb. Blood Flow Metab.* **20**, pp. 469–477, 2000.
10. V. Ntziachristos and B. Chance, "Probing physiology and molecular function using optical imaging: applications to breast cancer," *Breast Cancer Res* **3**, pp. 41–46, 2001.
11. R. Weissleder and V. Ntziachristos, "Shedding light onto live molecular targets," *Nature Medicine* **9**(1), pp. 123–128, 2003.
12. S. Arridge, "Optical tomography in medical imaging," *Inverse Problems* **15**, pp. R41–R93, April 1999.
13. A. P. Gibson, J. C. Hebden, and S. R. Arridge, "Recent advances in diffuse optical imaging," *Phys. Med. Biol.* **50**, pp. R1–R43, 2005.
14. L. V. Wang and H. Wu, *Biomedical Optics: Principles and Imaging*, John Wiley and Sons, Hoboken, New Jersey, 2007.
15. J. Chen and X. Huo, "Theoretical results on sparse representations of multiple measurement vectors," *IEEE Trans. Signal Process.* **54**(12), pp. 4634–4643, 2006.
16. O. K. Lee, J. M. Kim, Y. Bresler, and J. C. Ye, "Compressive diffuse optical tomography: non-iterative exact reconstruction using joint sparsity," *IEEE Trans. Med. Imag.* **30**(5), pp. 1129–1142, 2011.
17. D. P. Huttenlocher, G. A. Klanderman, and W. J. Rucklidge, "Comparing images using the Hausdorff distance," *IEEE Trans. Pattern Anal. Mach. Intell.* **15**, pp. 850–863, September 1993.