DIFFUSE OPTICAL TOMOGRAPHY USING GENERALIZED MUSIC ALGORITHM

Okkyun Lee[1], *Jongmin Kim*[1], *Yoram Bresler*[2] *and Jong Chul Ye*[1]

[1] Dept. of Bio and Brain Engineering, KAIST
373-1 Guseong-dong Yuseong-gu, Daejon 305-701, Korea

[2] Coordinated Science Lab, Univ. of Illinois at Urbana-Champaign
1308 W.Main Street, Urbana, IL 61801

ABSTRACT

Diffuse optical tomography (DOT) is an emerging imaging modality to reconstruct the optical parameters of a highly scattering medium. However, ill-posedness and nonlinearity of light scattering make the DOT problem very difficult. We showed that DOT problem can be formulated as a multiple measurement vector (MMV) problem, and compressive sensing approach like S-OMP can be used. However, for a limited number of illumination patterns, the conventional compressed sensing (CS) approach for DOT was not satisfactory. The main objective of this paper is to propose a new non-iterative and exact reconstruction algorithm for the diffuse optical tomography problem that outperforms the conventional compressive sensing approach, thanks to a recently invented generalized MUSIC algorithm. Simulation results confirm that the new algorithm outperforms the previous algorithms and reliably reconstructs optical inhomogeneities.

***Index Terms*—** Diffuse optical tomography, generalized MUSIC algorithm, multiple measurement vector problem, S-OMP

1. INTRODUCTION

Diffuse optical tomograph (DOT) is an emerging imaging modality that reconstructs optical parameters of a cross section of a highly scattering medium based on the scattered and attenuated optical flux measurement. Owing to its portability and sensitivity, DOT has emerged as an important tool for many biomedical imaging areas [1]. However, the inverse problem of DOT is highly nonlinear due to the nonlinear coupling between the unknown optical parameters and the photon flux. Furthermore, the problem is severely ill-posed due to the diffusive nature of light propagation. Conventionally, iterative or linearization methods have been proposed to deal with these difficulties, which is computationally expensive, or prone to error due to linearization.

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In many imaging scenarios, the optical parameter changes occupy only a small portion of the whole field of view (FOV). Furthermore, massive detector arrays are often used to measure the scattered optical flux from a limited number of illumination patterns. In this situation, the DOT problem can be converted to a multiple measurement vector (MMV) problem in a compressed sensing (CS) framework. Based on this, we proposed a non-iterative and exact reconstruction of DOT using a compressive sensing approach [2]. However, the performance of conventional MMV compressive sensing algorithms such as S-OMP or p-thresholding was shown to saturate as the number of illumination patterns increases [3]. On the other hand, the multiple signal classification (MUSIC) algorithm is better when sufficient illumination patterns are available, but its performance is limited when the number of illuminations is not sufficient [4].

The main contribution of this paper is a new type of non-iterative and exact reconstruction algorithm for diffuse optical tomography problem that outperforms all existing methods, thanks to a recently invented generalized MUSIC algorithm [5, 6]. The new algorithm is a hybrid approach that uses a conventional CS algorithm to find the support of $k-r$ targets, and uses a generalized MUSIC criterion to find the remaining r, where k is the number of targets and r is the number of independent illumination patterns. It has been shown that the performance of the generalized MUSIC algorithm approaches that of the MUSIC algorithm as the number of illumination patterns increases, and is reduced to a conventional CS algorithm for a single illumination case. Furthermore, the new algorithm is so flexible that it can reconstruct isolated targets as well as extended targets whose sparsity level is not well-defined. We showed the validity of the new algorithm for the DOT problem using extensive simulations.

2. PROBLEM FORMULATION

The photon flux density $u(\mathbf{x})$ from a continuous wave illumination $f(\mathbf{x})$ can be described by the following diffusion equation [1]:

$$\nabla \cdot D(\mathbf{x})\nabla u(\mathbf{x};l) - \alpha(\mathbf{x})u(\mathbf{x};l) = -f(\mathbf{x};l), \tag{1}$$

where $l = 1, \cdots, r$ denotes the specific illumination. Here, $\alpha(\mathbf{x}) = c\mu_a(\mathbf{x})$ and $D(\mathbf{x}) = c/3(\mu_a(\mathbf{x}) + \mu_s'(\mathbf{x}))$ are normalized absorption and diffusion coefficients, where $\mu_a(\mathbf{x})$ is the absorption coefficient, $\mu_s'(\mathbf{x})$ is the reduced scattering coefficient, and c is the speed of light, respectively. Assuming that the optical inhomogeneities are sparsely distributed at locations $\{\mathbf{x}_{(i)}\}_{i=1}^k$ selected from n possible locations $\{\mathbf{x}_i\}_{i=1}^n$, the scattered optical flux at $\mathbf{x}$ from the l-th illumination is given by

$$v(\mathbf{x}; l) = \sum_{i=1}^{k} g_0(\mathbf{x}, \mathbf{x}_{(i)}) u(\mathbf{x}_{(i)}; l) \Delta\alpha(\mathbf{x}_{(i)}), \qquad (2)$$

where $g_0(\mathbf{x}, \mathbf{x}_{(i)})$ is the homogeneous Green's function. By collecting the measurement of Eq. (2) at detector positions $\{\mathbf{x}_{d_i}\}_{i=1}^m$, the imaging problem can be represented in matrix form

$$Y = AX + E, \qquad (3)$$

where $Y \in \mathbb{R}^{m\times r}$ is the scattered flux , $E \in \mathbb{R}^{m\times r}$ is the noise matrix, $A \in \mathbb{R}^{m\times n}$ is the sensing matrix whose elements $A_{ij} = g_0(\mathbf{x}_{d_i}, \mathbf{x}_j)$ are samples of the Green's function, and $X \in \mathbb{R}^{n\times r}$ is the induced current matrix with $X_{ij} = u(\mathbf{x}_i; j)\Delta\alpha(\mathbf{x}_i)$ whose rows are nonzero when $\Delta\alpha(\mathbf{x}_j) \neq 0$. Then, the diffuse optical tomography problem can be stated as follows:

$$(P0): \quad \min ||X||_0, \quad \text{subject to} \quad ||Y - AX||_2^2 \leq \epsilon, \quad (4)$$

where $||X||_0$ denotes the number of non-zero rows of X. Denote by S the *active set* of non-zero rows of X. Then $(P0)$ can be addressed by first estimating S to satisfy (4), after which the corresponding X can be obtained using least-squares:

$$\hat{X}^S = A_S^{\dagger} Y, \qquad (5)$$

where the super-script † denotes the pseudo-inverse. Finally, $\Delta\alpha$ can be calculated from $\hat{X}^S$ using least square fitting without solving a differential equation [2]:

$$\Delta\alpha(\mathbf{x}_{(j)}) = \frac{\sum_{l=1}^{r} \hat{u}(\mathbf{x}_{(j)}; l)^* \hat{X}_{jl}^S}{\sum_{l=1}^{r} |\hat{u}(\mathbf{x}_{(j)}; l)|^2}, \quad j = 1, \cdots, k, \qquad (6)$$

where $\hat{u}(\mathbf{x}_{(j)}; l)$ is an optical flux estimated from $\hat{X}^S$ using the Foldy-Lax equation [2].

3. GENERALIZED MUSIC ALGORITHM

3.1. Theory

In some DOT problems, only a limited number of illuminations is allowed. For example, in neuro-imaging applications, due to the time-varying hemodynamics, we cannot increase the number of illuminations sufficiently without sacrificing temporal resolution. In this scenario, the source correlation matrix is not full-rank, hence the MUSIC based approach for DOT [7] is not accurate. On the other hand, compressive sensing approaches such as S-OMP [2] can be used for DOT problems when few snapshots is available. However, they become inferior to MUSIC as the number of available snapshots increases. Therefore, for the intermediate range of number of illuminations (i.e. $1 < r < k$), neither MUSIC nor S-OMP approaches are satisfactory. To address this, this paper employs a new class of algorithms called generalized MUSIC algorithm [5, 6].

More specifically, assume that $\text{rank}(Y) = r \leq k$ and the RIP (Restricted Isometry Property) condition $0 < \delta_{2k-r+1}^L(A) < 1$ is satisfied. Assume further that the non-zero rows of X are in general position in $\mathbb{R}^n$, $I_{k-r} \subset \text{supp}X$, $|I_{k-r}| = k - r$, $\text{supp}X$ is the true active set, and $Q \in \mathbb{R}^{m\times(m-r)}$ consists of orthonormal columns such that $Q^*Y = 0$, and $A_{I_{k-r}}$ denotes a submatrix of A collecting I_{k-r} column indices and $\mathbf{a}_j$ is the j-th column of A. The columns of Q are often called a basis for the "noise subspace". Then, we can show that the following holds [5, 6]

$$\mathbf{g}_j^* P_{G_{I_{k-r}}}^{\perp} \mathbf{g}_j = 0, \qquad (7)$$

if and only if $j \in \text{supp}X$, where $\mathbf{g}_j = Q^*\mathbf{a}_j$, $G_{I_{k-r}} = Q^*A_{I_{k-r}}$ and $P_{G_{I_{k-r}}}^{\perp}$ is the orthogonal projection on the ortho-complement of the range space of $G_{I_{k-r}}$. The criterion (7) is called generalized MUSIC criterion. This can be used to identify the remaining r-supports, when $k - r$ supports have been already estimated by other methods. Note that when $r = k$, the condition (7) coincides with the conventional MUSIC algorithm. For more details including noisy case analysis, see [5].

3.2. Generalized MUSIC Spectrum for Extended Targets

Since MUSIC was originally developed for continuous parameter estimation, the similarity of the generalized MUSIC criterion (7) to the original MUSIC criterion leads us a continuous form of the generalized MUSIC algorithm. This is important for DOT problem, where the optical inhomogeneities are continuously varying in space and the sparsity level k may be difficult to estimate. Indeed, the A matrix in (3) and sparsity level k are functions of the specific discretization scheme of the FOV, and the RIP constant also depends on the discretization interval. In general, if we choose a large grid interval, the initial estimate of the $k - r$ support is prone to discretization error. On the other hand, reducing the discretization interval has two deleterious effects. First, it increases the sparsity level k for a spatially distributed target of a given size without increasing r, thus increasing the number of error-prone $k - r$ steps of determining the partial support. Second, reducing the grid interval typically increases the mutual coherence between the atoms of the resulting sensing matrix A. The RIP constant of A usually increases as well, making recovery more difficult. The minimum grid interval can be determined by requiring that when propagated back to the measurements, the discretization error be below the noise level. Due to the diffusive nature of tissue, it was shown that two targets must be separated by more than $1 \sim 2mm$ to change the optical flux measurement at the detector location [8]. Hence, there is little if any advantage in choosing a much smaller grid interval, and we chose $0.5mm$ as the smallest grid interval to find the partial support. The remaining support can be found using a continuous form generalized MUSIC algorithm that does not depends on particular grid interval. The detailed algorithm is as follows.

- Step 1 : Calculate noise subspace of Q of Y.
- Step 2 : Define the discretization grid that matches the best possible resolution, and select a set I_{k-r} with $|I_{k-r}| = k - r$ using any MMV algorithm.
- Step 3 : Calculate $G_{I_{k-r}} = Q^* A_{I_{k-r}}$.
- Step 4 : Plot the generalized MUSIC spectrum $p(\mathbf{x})$

$$p(\mathbf{x}) = \frac{\mathbf{a}(\mathbf{x})^*\mathbf{a}(\mathbf{x})}{\mathbf{a}(\mathbf{x})^*\bar{P}^{\perp}_{G_{I_{k-r}}}\mathbf{a}(\mathbf{x})}, \ \mathbf{x} \in \Omega, \tag{8}$$

where
$$\bar{P}^{\perp}_{G_{I_{k-r}}} = QP^{\perp}_{G_{I_{k-r}}}Q^*, \tag{9}$$

and
$$\mathbf{a}(\mathbf{x}) = [g_0(\mathbf{x}_{d_1}, \mathbf{x}), \cdots, g_0(\mathbf{x}_{d_m}, \mathbf{x})]^T, \ \ \mathbf{x} \in \Omega.$$

4. NUMERICAL RESULTS

To validate the performance of the proposed method, we have conducted two experiments. First, isolated targets are reconstructed to quantify the superiority of the generalized MUSIC algorithm over original S-OMP. Second, an extended target is used for recovery to demonstrate the possibility of imaging spatially varying continuous functions with the generalized MUSIC spectrum.

4.1. Isolated target simulation

The simulation geometry for the isolated target is illustrated in Fig. 1(a). The field of view (FOV) is set to $3cm \times 3cm \times 3cm$. 3×3 detectors with a pitch of $1cm$ are located along each six surfaces of a cube as shown in Fig. 1(b). Line sources are used in this simulation. Within each experiment, we increased the number of sources progressively. Targets are regularly placed around the center the FOV with a distance of $\lambda = 7mm$ or $5mm$ as illustrated in Fig. 1(c). In each experiment we progressively increased the number of targets from the center while maintaining their distance of λ. The optical parameters for the homogeneous background are $\mu_a = 0.05cm^{-1}$, $\mu_s' = 7.5cm^{-1}$, whereas the absorption change is $\Delta\mu_a = 0.05cm^{-1}$. We added Gaussian noise of various SNRs to verify the noise robustness of our algorithm. The results were obtained by averaging 300 independent experiments, and the empirical recovery ratio is defined as the percentage of correct identification of all supports. In the proposed algorithm, $k - r$ support components are found using original S-OMP, and the remaining r components are found using the generalized MUSIC criterion.

Figs. 2(a) and (b) illustrate that even though the number of illuminations increases, S-OMP can recover only one target perfectly. The inferior performance is especially noticeable

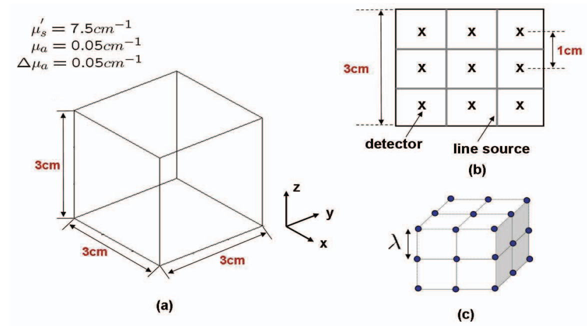


Fig. 1. Imaging geometry for isolated target simulation. (a) Field of view of the simulation, (b) detector and source configuration, and (c) the targets.

under noise. However, even when S-OMP failed to recover most of the targets, the overall reconstruction performance of the proposed method was significantly improved. Note that for $\lambda = 7mm$, the proposed algorithm has significant advantages over S-OMP. For $\lambda = 5mm$, the number of resolved targets is reduced. This is because the RIP degrades and the compressive sensing stage of reconstructing $k - r$ supports is not efficient. However, the MUSIC step still works fine and we have a significant gain over S-OMP. From Figs. 2(c)(d), we found that our algorithm is quite robust to noise – much more so than S-OMP.

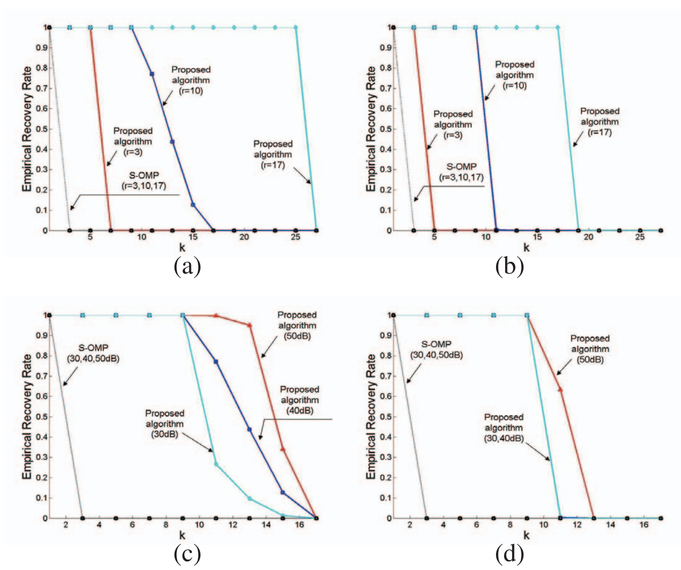
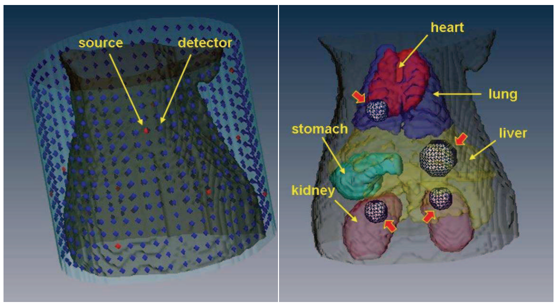


Fig. 2. Perfect reconstruction ratios for isolated target case with various number of illuminations $r = 3, 10$ and 17 when SNR $40dB$ for (a)(b), and with various SNR $30, 40$ and $50dB$ when $r = 10$ for (c)(d). λ is $7mm$ for (a)(c), and $5mm$ for (b)(d).

4.2. Extended target simulation

The source and detector geometry for the extended target simulation is illustrated in Fig. 3(a). We use 12 point sources and

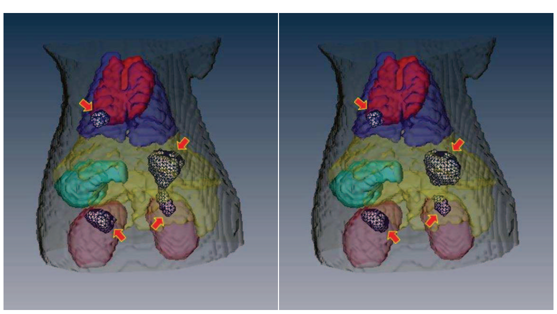
738 detectors placed on the surface of a cylinder, which is filled with matching fluid having the same optical parameter as the mouse skin. The radius and the height of the cylinder are $16mm$ and $32mm$, respectively. The mouse phantom is illustrated in Fig. 3(b) in which 4 tumors are indicated with red arrows, whereas the absorption change for the tumors is $\Delta\mu_a = 0.01mm^{-1}$. Gaussian noise of SNR $= 40dB$ was added to the scattered flux.



(a) (b)

Fig. 3. Imaging geometry for extended target simulation. (a) Source and detector geometry and (b) mouse phantom (the red arrows indicate tumors).

The sparsity level k can not be set *a priori*, since it depends on the grid size. Hence, as described in Section 3.2, we attempted to reconstruct inhomogenities using the generalized MUSIC spectrum. We chose a reconstruction grid size of $0.5mm$, and chose 516 support components using p-thresholding and assumed they corresponded to the $k-r$ support with the grid size of $0.5mm$. Then, the generalized MUSIC spectrum in Eq. (8) was calculated. Figs. 4(a)(b) illustrate the location of the thresholded spectra of the conventional MUSIC and the generalized MUSIC criterion, respectively, embedded in the original phantom. The rendered results show that reconstructed tumors in the liver and the nearby kidney are connected together into a big tumor for the case of the conventional MUSIC algorithm, whereas our algorithm clearly shows each tumor in the liver and kidney, separately.



(a) (b)

Fig. 4. 3D reconstruction of tumors embedded in the mouse phantom with (a) conventional MUSIC, and (b) generalized MUSIC criterion. The scattered flux is corrupted by additive Gaussian noise of SNR $40dB$.

5. CONCLUSION

This paper described a generalized MUSIC algorithm as a novel non-iterative and exact reconstruction algorithm for diffuse optical tomography. It is based on the observation that the DOT problem can be converted to a multiple measurement vector problem. The new algorithm overcame the limitation of conventional compressed sensing and the MUSIC algorithm. Furthermore, we showed that the new algorithm can be used to image extended targets using a generalized MUSIC spectrum. Extensive numerical simulations showed that the new algorithm is superior to previous algorithms.

6. REFERENCES

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