Sparse and Deep Learning Approaches for Biomedical Image Reconstruction: Part I: Compressed Sensing & Sparse Recovery

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This material can be downloaded from http://bispl.weebly.com
Applied mathematics, signal processing & machine learning for bio-medical imaging

MRI Acceleration
- Fast imaging using Compressed sensing
- ISMRM09 record challenge winner!
- k-t FOCSUSS

MRI Signal Processing
- MR artifact removal using ALOHA

Neuro-Imaging (statistical tool development)
- Diffuse optical tomography (DOT)
- Super-resolution imaging (FALCON)

Optical Imaging

PET Reconstruction
- Dynamic 4D PET reconstruction

Machine Learning/Deep Learning for Bio-medicine
- 2016 AAPM Low-Dose CT Grand Challenge Winner (WaveNet)

x-ray CT Reconstruction

Interior Tomography

http://bisp.kaist.ac.kr
Course Overview

1. Introduction
2. Part I: Compressive Sensing
   - Motion-compensated CS
   - Learning-based CS
   - Multiple measurement vector (MMV) CS
3. Part II: Low-rank Hankel Matrix Approaches
4. Part III: Deep learning approaches
5. Open problems and outlook
Roadmap: From CS to Deep Learning
Coherent Theme of Sparse Recovery
Resolution Limits in Medical Imaging

• **Diffraction Limit**

Resolution limit for optics, x-ray, PET, and etc.

\[ \text{Resolution} \approx \frac{\lambda}{2\text{NA}} \]

http://www.microscopyu.com/
Resolution Limits in Medical Imaging

• Nyquist Sampling limit

- Nyquist requires the minimum two times sample rate w.r.t. signal bandlimit
- Resolution limit for Fourier based imaging (MRI, etc.)
Law of Parsimony: Sparsity, Low-rank, LDM

- Occam’s Razor: law of parsimony by William Occam (14th century)
  
  *Entities should not be multiplied beyond necessity.*
- All things being equal, the **simplest** solution tends to be the best one.
Super-resolution Microscopy

The Nobel Prize in Chemistry 2014

Eric Betzig
Prize share: 1/3

Stefan W. Hell
Prize share: 1/3

William E. Moerner
Prize share: 1/3

Compressed Sensing MR Imaging

- Forward problem

- Sparse recovery problem

Figure courtesy of Mathews Jacob
9 View CT Reconstruction using Deep Learning
COMPRESSED SENSING
Compressed Sensing (CS)

- Incoherent projection
- Underdetermined system
- Sparse unknown vector

\[ b = A x \]

\[ m \times 1 \text{ measurements} \]

\[ m \approx k \log(n) \ll n \]

\[ n \times 1 \text{ vector} \]

\[ k \text{ # non-zeros} \]

Courtesy of Dr. Dror Baron
Min-norm Solution is Not sparse

\[ T = \{x_p\} + \mathcal{N}(A) \]
$L_0$ Minimization

\[
\min_x \|x\|_0 \\
\text{subject to } b = Ax
\]

$\|x\|_0 = \{\# \text{ of non-zero elements of } x\}$

- Two fundamental questions
  - Uniqueness ?
  - Convex relaxation ?
Uniqueness and Spark

**Definition**

Given a matrix $A = [a_1, \cdots, a_n]$, the quantity $\text{spark}(A)$ is the smallest possible number of linearly dependent columns of $A$.

**Theorem (Donoho, Elad)**

$x \in \mathbb{R}^n$ is the unique solution of the problem $[P0]$ if $Ax = b$ and

$$\|x\|_0 < \frac{\text{spark}(A)}{2}.$$
L1 Convex Relaxation

\[
\begin{align*}
\min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 \\
\text{subject to} \quad & \mathbf{b} = A\mathbf{x} \\
\end{align*}
\]

\[\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|\]
Figure 2: Estimation picture for the lasso (left) and ridge regression (right)

Sufficient Conditions for P1

**Theorem (Donoho, Huo)**

If \( x \in \mathbb{R}^n \) is a \( k \)-sparse vector such that \( Ax = b \) and

\[
k < \frac{1}{2} \left( 1 + \frac{1}{\mu(A)} \right)
\]

or

\[
k < \mu_{1/2}(A)
\]

then \( x \) is the unique solution to \([P1]\) and this solution is identical with the solution to \([P0]\).

**Theorem (Strohmer, Heath)**

Let \( A \in \mathbb{R}^{m \times n} \) with normalized columns. The mutual coherence must satisfy

\[
\mu(A) \geq \sqrt{\frac{n - m}{m(n - 1)}}.
\]

Note: In the deterministic setting, we must obtain at least \( O(k^2) \) samples.
RIP Conditions

- RIP has been developed for probabilistic argument.

**Definition**
A matrix $A \in \mathbb{R}^{m \times n}$ is said to have a $k$-restricted isometry property (RIP) if there is a constant $0 < \delta_k < 1$ such that

$$(1 - \delta_k)\|x\|^2 \leq \|Ax\|^2 \leq (1 + \delta_k)\|x\|^2 \quad [RIP]$$

for every $k$-sparse vector $x$.

**Theorem (Candes)**

If $x \in \mathbb{R}^n$ is a $k$-sparse vector such that $Ax = b$ and $A$ satisfies the RIP condition with

$$0 < \delta_{2k} < 1$$

then $x$ is the unique solution to $[P0]$. 
RIP as Sufficient Condition

Theorem (Candes)

Let $\mathbf{x} \in \mathbb{R}^n$ be a $k$-sparse vector such that $A\mathbf{x} = \mathbf{b}$. If $A$ satisfies the [RIP] with $\delta_{2k} < \sqrt{2} - 1$ then

- there is a unique $k$-sparse solution $\mathbf{x}$ consistent with $\mathbf{b}$;
- the problem $[P1]$ has a unique solution;
- the solution to the $[P1]$ is equal to $\mathbf{x}$. 
Suppose that the measurements are corrupted by bounded noise so that

\[ \mathbf{b} = A\mathbf{x} + \mathbf{n} \]

where \( \|\mathbf{n}\| \leq \epsilon \). In order to recover \( \mathbf{x} \), we use the modified \( L_1 \)-problem

\[ \min \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{b} - A\mathbf{x}\| \leq \epsilon. \quad [P1'] \]

**Theorem (Candes)**

*Let \( \mathbf{b} \) is the noisy measurements of \( \mathbf{x} \in \mathbb{R}^n \) with noise \( \|\mathbf{n}\| \leq \epsilon \). Let \( \mathbf{x}^k \) denote the best \( k \)-sparse approximation of \( \mathbf{x} \) and \( \mathbf{x}' \) be the solution of \([P1']\). If \( A \) satisfies the RIP with \( \delta_{2k} < \sqrt{2} - 1 \), then*

\[ \|\mathbf{x} - \mathbf{x}'\| \leq \frac{C_1}{\sqrt{k}} \|\mathbf{x} - \mathbf{x}^k\|_1 + C_2 \epsilon \]

*for some constants \( C_1, C_2 > 0 \).*
Sparse Recovery Formulations

- **Analysis Formulation**

\[
\begin{align*}
\min_{\mathbf{x}} & \quad \|T\mathbf{x}\|_1 \\
\text{subject to} & \quad \|\mathbf{b} - A\mathbf{x}\| \leq \epsilon
\end{align*}
\]

- **Synthesis Formulation**

\[
\begin{align*}
\min_{\mathbf{\theta}} & \quad \|\mathbf{\theta}\|_1 \\
\text{subject to} & \quad \|\mathbf{b} - A\Psi\mathbf{\theta}\| \leq \epsilon
\end{align*}
\]
Application: Cardiac MRI

- Balanced SSFP

Sequence diagram of b-SSFP

![Sequence diagram of b-SSFP](image)

FLASH  

b-SSFP

- Demanding Acquisition Requirements

<table>
<thead>
<tr>
<th>Matrix size</th>
<th>TR (msec)</th>
<th>Views per Segment</th>
<th>Temporal Resolution</th>
<th>Spatial Resolution</th>
<th>Total Acq. time/slice</th>
<th>Total Acq. time/volume</th>
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<tbody>
<tr>
<td>100x256</td>
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<td>15</td>
<td>60 msec</td>
<td>2.5mm × 1.4mm</td>
<td>7 sec</td>
<td>56 sec</td>
</tr>
<tr>
<td>196x256</td>
<td>4</td>
<td>15</td>
<td>60 msec</td>
<td>1.3mm × 1.4mm</td>
<td>13 sec</td>
<td>104 sec</td>
</tr>
</tbody>
</table>

An example of Cartesian TrueFISP parameter for cardiac imaging. The acquisition time is calculated based on 60bpm patients with 8 slice acquisition. Flip angle of 40-60° is commonly used.

Aliasing in accelerated Dynamic MRI

Down sampling along k-t space

➔ Aliasing artifact
Classical Methods

• Parallel imaging (SENSE, GRAPPA)
• k-t space method
  – UNFOLD
  – UNFOLD-SMASH, TSENSE
  – K-t BLAST/SENSE
  – PARADISE, PARADIGM

Eight aliased image with different sensitivity

Blaimer et al, Top Magn Reson Imaging • V15, N 4, August 2004
How to Improve Sparsity!!

Cardiac MR

fMRI
RIP in Dynamic MR

\[ y = \Lambda x = \Phi \psi x \]

- **k-t space sample**
- **Random phase encoding**
- **Temporal transform**
k-t FOCUSS

(Jung et al, PMB:2007, MRM:2009(a), MRM:2009(b))

**k-t FOCUSS (our method)**

\[ x_{n+1} = x_0 + \Theta_n A^H (A\Theta_n A^H + \lambda I)^{-1}(y - Ax_0) \]

\[ \Theta_n = W_n^H W_n = \text{diag}(|x_{n,1}|^{2p}, \cdots, |x_{n,N}|^{2p}), \quad 0.5 \leq p \leq 1 \]

**Optimal from compressed sensing perspective**

When there is no update, **k-t FOCUSS** is exactly same with **k-t BLAST/SENSE**

**k-t BLAST/SENSE (J. Tsao, MRM, 2003)**

\[ x = \bar{x} + \Theta_0 A^H (A\Theta_0 A^H + \lambda I)^{-1}(y - A\bar{x}) \]

\[ \Theta_0 = \text{diag}(|x_{n,1}|^2, \cdots, |x_{n,N}|^2) \]

Does not solve **L1 minimization** problem

Not Optimal from **CS** perspective
11 x accelerated
Zero-padding inverse FFT from measurements

Sampling pattern
11 x accelerated

k-t BLAST

Sampling pattern
11 x accelerated k-t FOCUSS with temporal average

Sampling pattern
MOTION-COMPENSATED COMPRESSED SENSING
### Lessons from HDTV History

<table>
<thead>
<tr>
<th>MUSE</th>
<th>MPEG</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first HDTV (Japan) (obsolete)</td>
<td>Modern Standard (MPEG1,2,4..)</td>
</tr>
<tr>
<td>quincunx sampling</td>
<td>ME/MC</td>
</tr>
</tbody>
</table>

* MUSE: MUltiple Sub-nyquist Encoding systems

Figures from Kovacevic et al, IEEE TIP, 1993

- Lattice sampling theory → Low compression
- ME/MC + residual coding → High compression
# MUSE vs k-t MR

<table>
<thead>
<tr>
<th>MUSE</th>
<th>k-t MR</th>
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<tr>
<td>The first HDTV (Japan) (obsolete)</td>
<td>UNFOLD</td>
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<tr>
<td>quincunx sampling</td>
<td>k-t BLAST/SENSE</td>
</tr>
<tr>
<td>Lattice sampling theory</td>
<td>k-t space downsampling</td>
</tr>
<tr>
<td>➔ Low compression</td>
<td>➔ Low acceleration !!</td>
</tr>
</tbody>
</table>

*Figures from Kovacevic et al, IEEE TIP, 1993*

* MUSE: MULTiple Sub-nyquist Encoding systems
Reference is often Free

• Cardiac MR
  – During volume acquisition, diastole phase can be acquired

• fMRI

• MR angiography, vocal tract, etc.
  • example) RIGR, SPEAR, etc.
Random Sampling with Reference

We want to distribute k-space samples to allow prediction and residual coding.

**When two reference frames can be obtained,**

**When one reference frames can be obtained**

\[ x_0 \]
ME/MC in MPEG Video

*Full sampled measurements*

*Encoder*
ME/MC in MPEG Video

Motion Estimation (ME) search area
Matched block: has the smallest MAD
current pixel
block: size N
position: (x, y)

I: reference
P: dynamic frame

Motion vector: (i, j)

[x, y]

[I P P P P P P P I]
ME/MC in MPEG Video

Residual encoding
ME/MC in Dynamic MRI

- Encoder
- Down sampled measurements

**ME is not possible!!**
Reinable Motion Estimation
Refinable Motion Estimation

Initialization with k-t FOCUSS
Refinable Motion Estimation

Initialization with $k$-$t$ FOCUSS
Refinable Motion Estimation

Apply ME/MC
Refinable Motion Estimation

Residual Encoding using k-t FOCUSS

\[ x_{n+1} = \bar{x} + \Theta_n A^H \left( A\Theta_n A^H + \lambda I \right)^{-1} (y - A\bar{x}) \]
Cine MRI

• Acquisition parameters
  - 1.5 T Philips scanner at Yonsei University Medical Center in Korea
  - bSSFP
  - FOV = 345.00 x 270.00 mm²
  - Matrix size: 256 x 220 (Read-Out x PE)
  - TR = 3.45ms
  - flip angle: 50°
  - heart rate: 66 bpm
  - # of cardiac phases: 25 frames

• Among fully sampled k-data, partial k-data were arbitrarily chosen.
11 x accelerated
Zero-padding inverse FFT from measurements

Sampling pattern
11 x accelerated

k-t BLAST

Sampling pattern
11 x accelerated
k-t FOCUSS

Sampling pattern
Radial Acquisition

iRadon 16 views

Radial k-t FOCUSS with ME/MC from 16 views
World-first In vivo Experiment
(Joint work with K. Nayak at USC)

- Acquisition parameters
  - 3.0 T GE Signa EXCITE at the University of Southern California
  - bSSFP
  - FOV = 300 x 300 mm²
  - Matrix size : 240 x 256 (PE x Read-Out)
  - TR = 3.7ms
  - views per segment = 10
  - flip angle : 45°
  - heart rate : 65 bpm
  - # of cardiac phases : 25 frames
  - acquisition time for 6 x accelerated data = 4 heart beats
  - acquisition time for fully sampled data = 24 heart beats
In Vivo Experiment (6x accel.)

Zero-padding inverse FT from measurements

k-t FOCUSS with ME/MC

full data
Which One Is Optimal?

FT along t-axis
PCA along t-axis
WVT along x-axis + FT along t-axis
KLT (PCA): optimal transform

Karhunen Loeve Transform is well known for an optimal sparsifying transform of general signals.

KLT basis: Matrix of Eigen vectors for $XX^T$

Considering SVD

$X = U \Sigma V^T$

$\Sigma = \begin{pmatrix} L & 0 \\ 0 & s \end{pmatrix}$

$L$: Large Singular values
$s$: small Singular values
$L >> s \rightarrow 0$

Hence, KLT

$U^T X = \Sigma V^T$

is SPARSE
PCA in k-t FOCUSS

\[ x_{n+1} = x_0 + \Theta_n A^H (A\Theta_n A^H + \lambda I)^{-1} (y - Ax_0) \]

Random phase encoding

k-t space sample

Temporal transform

Significant eigenvector

\[ y = Ax = \Phi \Psi x \]
Two step Boosting for PCA basis
MR Angiography

Golden Section Radial Trajectory

Target time points: 39, 79, 118, 157, 197, 236, 275, 314, 354, 393, 432, 472

Measurement size (512 read-out x 512 views) vs Total recon matrix (512 x 512 x 12 time frames)

12 x acceleration !!
KLT for CE-MRA

k-t FOCUSS recon using FFT

Summation along t-axis

Thresholding
2009 ISMRM Recon Challenge Award

L2 minimization  k-t FOCUSS using KLT
T2 Parameter Mapping

(joint work D. Kim, NYU: Feng et al, MRM, 2011)

\[ M_{xy}(t) = M_{xy}(0)e^{-t/T_2} \]

Detect T2 changes due to variety of disease
Cardiac T2 Mapping

6 x accel.  Conventional method  6 x accel.  k-t FOCUSS
Cardiac T2 Mapping

1.8 x accel.  GRAPPA  6 x accel.  k-t FOCUSS
T2 Estimation

\[ S(t) = \left( S_{ideal}^2 + \sigma^2 \right)^{1/2} \quad S_{ideal} \equiv S_0 e^{-t/T_2} \]
MULTIPLE MEASUREMENT VECTOR
COMPRESSED SENSING
MMV: Multiple Measurement Vector Problems

minimize $\|X\|_0$ subject to $B = AX$.

$k = \|X\|_0$ : number of nonzero rows
$r :$ number of snapshots
$m :$ number of sensor elements

Here, we assume that the columns of $B$ are linearly independent.
Inverse Scattering Problems

**Geophysical**
- Electromagnetic
- Acoustic

**Medical**
- Ultrasonic
- Optical

**Industrial**
- Electromagnetic
- Ultrasonic
- Optical

Slide courtesy by Devaney
Diffuse Optical Tomography

Near-infrared (NIR, ~650-950 nm)

* Durduan et al., MICCAI, 2010
Elastic Source Imaging

Elastography: Medical, geophysical applications

https://iame.com/online/breast_elastography/

https://marcellusdrilling.com/2018/04/9-more-seismic-testing-devices-stolen-in-swpa-6-were-returned/
Multiple Measurement Vectors from Multiple illumination Patterns

Finite # of snapshots are available
Sparse Signal \rightarrow \text{Perturbations in the optical parameters}

*Angiogenesis in cancer*
Inverse Scattering Problems

\[ \psi^{(in)}(r; s_0) = e^{ik s_0 \cdot r} \]

\[ \psi = \psi^{(in)} + \psi^{(s)} \]

**Existing methods**

- Born approximation \( \rightarrow \) linearization error
- Iterative Born approximation \( \rightarrow \) computationally expensive
Overcoming Nonlinearities using Joint Sparsity

\[ u^S_m(x) = -\frac{1}{D_0} \int_{\bigcup_{n=1}^N \Omega_n} G^{x_0}(x, y) u^t_m(y) f(y) dy, \]
L₀ Uniqueness of MMV

**Definition**

Given a matrix $A$, let $\text{spark}(A)$ denote the smallest number of linearly dependent columns of $A$.

**Theorem (Rao / Chen, Huo / Feng, Bresler / Davies, Eldar)**

If a matrix $X$ satisfies $AX = B$, then

$$
\|X\|_0 < \frac{\text{spark}(A) + \text{rank}(B) - 1}{2}
$$

if and only if $X$ is the unique solution to the $l_0$ minimization problem.

*With increasing number of snapshots, more non-zero elements can be recovered.*
Spark Reduction Principle


Theorem

Let \( r \leq m < n \). Suppose that we are given a sensing matrix \( A \in \mathbb{R}^{m \times n} \) and an observation matrix \( B \in \mathbb{R}^{m \times r} \). If the \( k \) nonzero rows of \( X \) are in general position (i.e., any collection of \( r \) nonzero rows are linearly independent) and \( A \) satisfies the RIP condition \( 0 \leq \delta_{2k-r+1}^{L}(A) < 1 \), then

\[
\text{spark}(Q^*A) = k - r + 1.
\]

Note that \( A \in \mathbb{R}^{m \times n} \) satisfies RIP with \( 0 \leq \delta_{2k-r+1}^{L}(A) < 1 \) if and only if

\[
k < \frac{\text{spark}(A) + \text{rank}(B) - 1}{2}.
\]

: \( l_0 \) bound for MMV problem

CS-MUSIC achieves \( l_0 \) bound with finite snapshot.
Theorem

Assume that we have an MMV problem $AX = B$, where $A$, $X$ and $B$ as in the previous theorem. If $I_{k-r} \subset \text{supp}X$ with $|I_{k-r}| = k - r$ and $A_{I_{k-r}} \in \mathbb{R}^{m \times (k-r)}$ which consists of columns of $A$, whose indices are in $I_{k-r}$, then for any $j \in \{1, \cdots, n\} \setminus I_{k-r}$,

$$a_j^* \left[ P_R(Q) - P_R(P_R(Q)A_{I_{k-r}}) \right] a_j = 0 \iff j \in \text{supp}X.$$  

$$a_j^* P_{[A_{I_{k-r}}B]} a_j = 0 \iff j \in \text{supp}X.$$  

Augmented Signal Subspace: $R[A_{I_{k-r}}B]$
Geometric Interpretation

\[ R(P_{R(Q)} - P_{R(QQ^*A_{I_{k-r}})}) \]

(noise subspace for generalized MUSIC)

\[ R(Q) \]

(noise subspace for MUSIC)

\[ R(QQ^*A_{I_{k-r}}) \]

\[ R(B) \]

(signal subspace)

\[ a_j \quad (j \in S) \]

\[ R(A_S) \]

\[ R(A_{I_{k-r}}) \]
Sampling Rate for MMV

Case 1: $r$ is a fixed number

Theorem

Let $\frac{\sigma_k([A_{k-r} S])}{\Delta} > 1 + \frac{k}{r}$. If we have

$$m > \frac{1 + \delta}{1 - 2\frac{k}{r} \eta_{k-r}} \frac{2k \log (n - k)}{r},$$

then we can find $k - r$ correct indices of $\text{supp} \ X$ by applying subspace S-OMP.

1. [Fletcher, Rangan] For SMV, when $\text{SNR} \to \infty$, we need $m > 2(1 + \delta) k \log (n - k)$ for some $\delta > 0$.

2. The sampling ratio is reciprocally proportional with respect to the number of multiple measurement vectors.

3. $\text{SNR} \to \infty$ is required, in the large system limit. (Similar to Reeves and Gaspar)
Sampling Rate for MMV

Case 2: $\alpha := \lim_{n \to \infty} \frac{r(n)}{k(n)} > 0$

Theorem

Let $\frac{\sigma_k([A_{I_{k-r}} S])}{\Delta} > 1 + \alpha$ Then if we have

$$m > (1 + \delta)^2 \frac{k}{1 - 2^{\frac{\eta_{k-r}}{\alpha}}} [2 - F(\alpha)]^2,$$

for some $\delta > 0$ where $F(\alpha)$ is an increasing function on $[0, 1]$ such that $F(1) = 1$ and $\lim_{\alpha \to 0+} F(\alpha) = 0$. Then we can find $k - r$ correct indices of suppX by applying subspace S-OMP.

1. If $\alpha \to 1$ and SNR $\to \infty$, then we only need to have $m > (1 + \delta)k$ for a small $\delta > 0$. (cf. MUSIC)
2. In this case, the required number of sensor elements is at most $4(1 + \delta)k$ when SNR$\to \infty$ (No log $n$ factor).
3. The required SNR is finite.
MMV for Bio-medical Imaging

- Parallel MRI + CS
- EEG/MEG
- Diffuse optical tomography
- Wave inverse scattering
ODT under Born/Rytov Approximation

Sample

x(y)

Scattered field 1

Scattered field 2

Z

2-D Fourier transform and scaling

3d k-space

k_x

k_y

(a)

k_x/\lambda

k_y/\lambda

(b)

k_x

k_y

(c)
ODT using Joint Sparsity

Lippman-Schwinger equation

\[ U_s(r) = \tau K_D[U](r) := \tau \int_D k^2 f(r') G(r,r') U(r') dr', \quad r \in \Omega, \]

MMV for Support & Induced Current Recovery

\[
\begin{align*}
U_{s}^{(m)}(r) &= \tau k^2 \int_D G(r,r') I^{(m)}(r') dr', \quad r \in \Gamma, \\
I^{(m)}(r) &= f(r) U^{(m)}(r), \quad m = 1, \ldots, M, \\
\min_I \|I\|_0, \quad \text{subject to } \|Y - G \cdot I\|_F \leq \epsilon.
\end{align*}
\]
ODT using Joint Sparsity

Unknown Total Field Estimation

\[ \hat{U}(r) = U_0(r) + \tau \kappa^2 \int_D G(r, r') \hat{I}(r') \, dr', \quad r \in D, \]

Abnormality Estimation

\[ U_s(r) = \tau \kappa^2 \int_D G(r, r') \hat{U}(r') f(r') \, dr', \quad r \in \Gamma. \]
Super-Resolution from Joint Sparse Recovery

(Lim et al, OPEX, 2017)

\[ u_s(r) = \sum_i a_i (1/\tau' - K_{D'})^{-1} K_{D'} [\phi_i](r) = \sum_i a_i \frac{\lambda_i}{1/\tau' - \lambda_i} \phi_i(r), \quad r \in D', \]
Super-Resolution from Joint Sparse Recovery
(Lim et al, OPEX, 2017)
Super-Resolution from Joint Sparse Recovery
(Lim et al, OPEX, 2017)
Diffuse Optical Tomography Applications

(Lee, OPEX, 2013)

- Line source shifting with 2 mm space
- 30 mm
- 20 x 20 point detectors (1.5 mm pitch)
- 22 mm
- 38 mm

<table>
<thead>
<tr>
<th>Optical parameters in mm$^{-1}$</th>
<th>$\delta \mu_a$</th>
<th>$\delta \mu'_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target 1</td>
<td>0.0504</td>
<td>0.8444</td>
</tr>
<tr>
<td>Target 2</td>
<td>0.0423</td>
<td>0.0</td>
</tr>
<tr>
<td>Target 3</td>
<td>0.0</td>
<td>0.8619</td>
</tr>
</tbody>
</table>
Diffuse Optical Tomography Applications

(Lee, OPEX, 2013)
Summary: Part I

- Sparsity principle is very important for biomedical image reconstruction
- Compressed sensing has many extensions
  - Compressed sensing
  - Motion compensated CS
  - Learning-based CS
  - MMV CS
- Downside: suffers from discretization, RIP, optimization bottleneck
References: Part I


• Jung, H., Park, J., Yoo, J., & Ye, J. C. (2010). Radial k-t FOCUSS for high-resolution cardiac cine MRI. Magnetic Resonance in Medicine, 63(1), 68-7
References: Part I


Min, J., Jang, J., Keum, D., Ryu, S. W., Choi, C., Jeong, K. H., & Ye, J. C. (2013). Fluorescent microscopy beyond diffraction limits using speckle illumination and joint support recovery. Scientific Reports, 3(2075)


References: Part I


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